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Mathematics 8

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Unit 7

Data Analysis and Probability

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Mathematics 8

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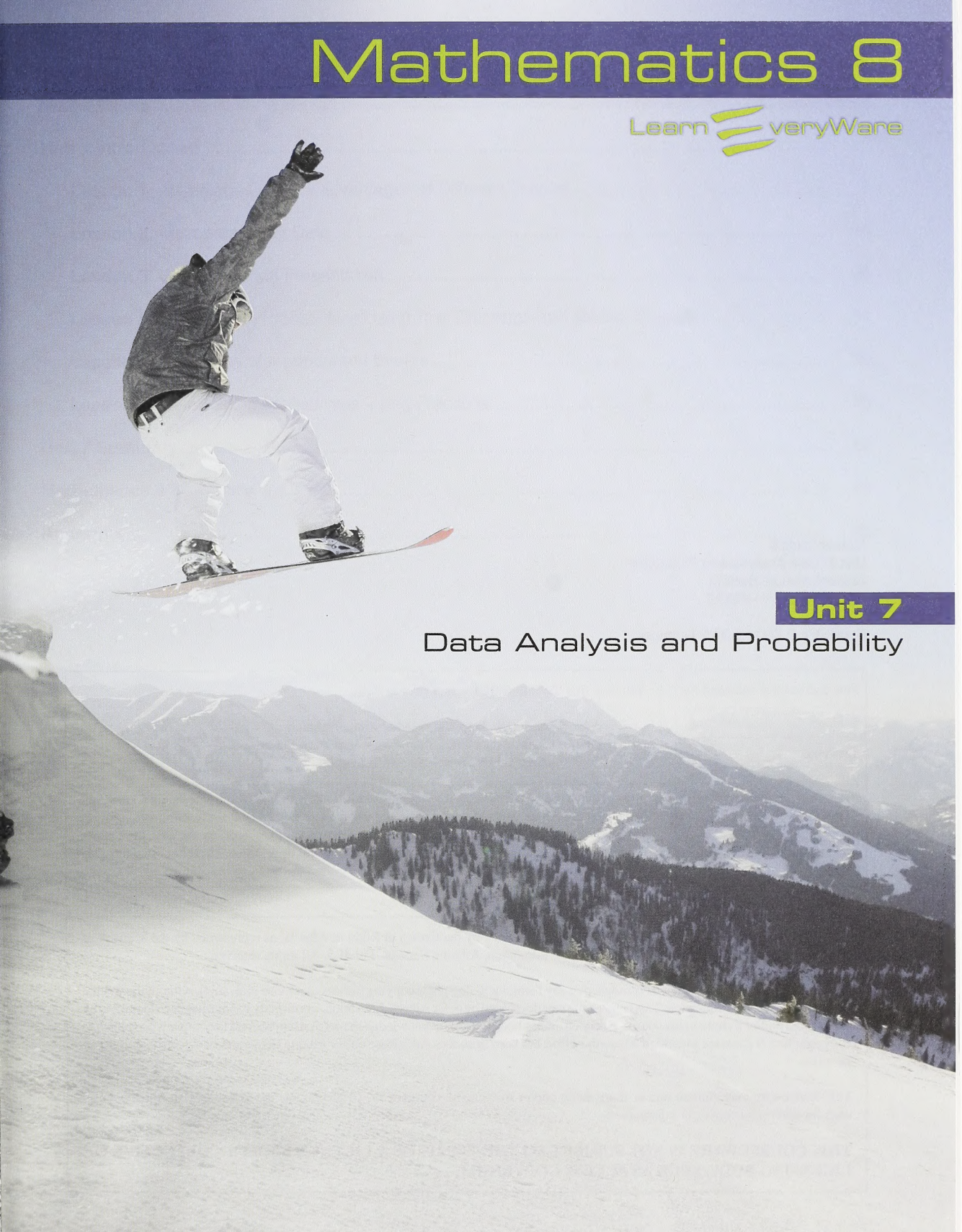

Unit 7

Data Analysis and Probability

Mathematics 8
Unit 7: Data Analysis and Probability
Student Module Booklet
ISBN 978-0-7741-3137-7

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Teachers	✓
Administrators	
Home Instructors	
General Public	
Other	

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- Learning Resources Centre, **http://www.lrc.education.gov.ab.ca**
- Tools4Teachers, **http://www.tools4teachers.ca**

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Unit 7: Data Analysis and Probability

Unit 7 Introduction

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Based on what you know about family members, classmates, and friends, you are likely aware that people like to spend their free time in different ways. Watching professional sports, listening to music, and playing tabletop games are a few popular pastimes.

Many of these pastimes involve data—information presented in numerical form. If you follow the music or sports scene, you need to be able to read data. Music sales by artists are often given in data tables of numbers. If you turn to a sports section of a newspaper, you will see data tables of numbers describing the rankings of athletes and team standings in a variety of sports.

Other pastimes involve some form of chance, which is also known as probability (i.e., the likelihood of an event) or luck.

If you plan to go outside to see your favourite team play, you may want to listen to the weather report to find out the probability of precipitation—the possibility of rain may affect your plans. If you stay indoors and play a tabletop game that uses dice or cards, probability affects each turn of the game. An understanding of probability will give you an advantage in a variety of situations.

In this unit you will investigate the use of graphs to represent data and develop strategies to determine the probability of events.

As you progress through this unit, you will gain the knowledge and skills needed to make graphs and display data related to music sales and to design a game using concepts of probability.

This unit will help you answer the following critical questions:

- How can you use graphs to represent and communicate data?
- How do you determine the probability of events?

In this unit you will use a handheld calculator or a computer for the display of data and the calculations of probability, conduct research using the Internet, and identify techniques used by the media to draw out particular responses from an audience.

The new concepts and skills relating to representing data and determining probabilities will be presented in six lessons.

Mathematics 8 Learn EveryWare

Unit 7: Data Analysis and Probability

Lesson 1: Advantages and Disadvantages of Different Graphs

Lesson 2: Misrepresenting Data

Lesson 3: Critiquing Data Presentation

Lesson 4: Determining Probabilities Using Tree Diagrams and Tables

Lesson 5: Outcomes of Independent Events

Lesson 6: Determining Probabilities Using Fractions

In this unit there are a variety of assignments you will be asked to complete. Some of these include:

- interacting with other students through a discussion board
- adding samples of your work to your Math 8 folder
- completing sets of questions for each lesson
- completing two unit problems presented at the end of the unit
- possibly writing a unit test

Strategies for Success

In order to support your success in this unit, follow these strategies.

Strategy 1

Make foldable study tools according to the detailed instructions on pages 4 and 408 of your textbook. There are two foldable study tools for you to create in this unit. Although these activities may not be graded for marks, you will benefit from having these tools as you complete practice activities and prepare for evaluation.

- As you are working through the lessons, add formulas, examples, and vocabulary words.
- Each foldable can serve as a quick reference guide and will help you save time when you are ready to study for a unit test or to work on your unit problem.

Strategy 2

In this unit you will be referring two chapters in your textbook—Chapter 1 and Chapter 11. Chapter 1 spans pages 2 to 41 of your textbook, and Chapter 11 is on pages 406 to 441.

- Take time to flip through the pages of each of these chapters in your textbook.
- View the introductions to the two textbook chapters and make a comparison of what each chapter is about. Can you identify which of the two chapters is more about recording events in the past and which is more about predicting likelihood of events still to happen?
- Look at all illustrations, margin features, and main titles to get a sense of what you will be studying. The key words may also help you get a sense of the chapters' content.

Unit 7 Problem

There are two problems to complete for this unit:

- In Unit Problem 1, Notes on Music, you will prepare a school newspaper article about a topic related to the music industry.

- In Unit Problem 2, A Fair Chance, you will analyze a probability game and improve it so that the scoring is fair to each player.

In the Unit Summary you will find more information about the unit problems. If you look ahead to the Unit Summary, you may be able to complete some portions of the unit problems as you work on certain lessons. You will also gain a better idea of how to schedule your time to complete these problems. The unit problems will likely seem less overwhelming if you complete the problems as you work through the lessons.

In Grade 7 math, you studied the mean, medium, and mode of data and the use of circle graphs to represent data. You also conducted some probability experiments and compared their results with theoretical probabilities. In this unit you will build on your earlier study of data and probability.

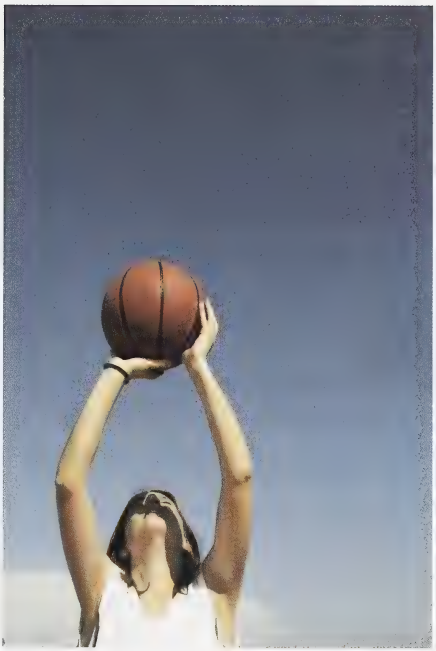
So, get going to begin the lessons of this unit!

Unit 7: Data Analysis and Probability

Lesson 1: Advantages and Disadvantages of Different Graphs

Get Focused

Having a good reach is an advantage in the game of basketball. Many basketball players are tall. Yet, on any winning team, players do vary in height.



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Turn to page 6 in your *MathLinks 8* textbook. On the chalkboard in the photograph, the students have collected information about the heights of their classmates. It is clear that there is variety in heights in the group measured.

The textbook leaves you with a question about the most common height for the boys and the girls in your class. In order to answer this question, you will need the height of each of your classmates. The height information you need is given to you in the Explore section of the lesson, and you will use graphs to analyze the height information.

The group that you know through the Math 8 discussion board may be considered to be students in your class.

In this lesson you will present data using different types of graphs. Then, you will critically examine the different graphs to identify the advantages and disadvantages of each graph. This will help you to select an appropriate graph to effectively present data in a meaningful way.

This lesson will help you answer the following critical question: How do you choose the type of graph to use when representing data visually?

In this lesson you may need the following:

- chalk, masking tape, or other marker
- a metre-stick, a large book, or other flat surface (e.g., used to extend from top of head to chalkboard, bulletin board, or other vertical surface that you can mark)
- a measuring tape

You may also need the following, which are available on the Math 8 Multimedia DVD:

- "Percent Circles"
- "0.5 Centimetre Grid Paper"
- "1-cm Grid Paper"
- "Frequency Table Template"
- "Graphing Height of Students"
- "Compare a Bar Graph and a Pictograph"

Assignments

Your assignment will consist of the following:

- posting to your discussion board
- adding to your Math 8 folder
- completing Unit 7: Lesson 1 Question Set

Explore

In the "Explore the Math" activity presented on pages 6 and 7 of the textbook, you will collect the height measurements of other Grade 8 students. You will compare different ways of graphing the data related to the height of other students.

You will need data for 12 students, which include males and females. You may wish to collect the height measurements of the students on your discussion board in this course. **Note:** Ask your teacher to supply sample height data for this Explore activity if you are unable to collect the data due to your learning environment.

You will need to fill in a data table like the following:

Heights of Students		
Student	**Gender**	**Height (cm)**
Alfredo	M	
Cora	F	

If possible, include your own height. To have your height measured, ask a learning partner to help you. Ask your partner to hold something flat—for example, a metre-stick or book—horizontally from the top of your head to a wall or chalkboard and to mark your height on the wall or a chalkboard.

The photo on page 6 of your textbook shows you how to mark the height of a person. Use a measuring tape to measure the height of the mark on the wall.

Try This

TT 1. Complete "Explore the Math" question 2 on page 6. Remember to ask your teacher if you need height data in addition to the measurements that you have been able to collect.

TT 2. Complete "Explore the Math" question 3 on page 6. Use the intervals 110–119, 120–129, and so on to make a frequency table. You may print out the blank "Frequency Table Template" from the Math 8 Multimedia DVD to make a tally of your results and to record the frequencies for each interval.

If any of the provided links take you to the "Mathematics Glossary" home page rather than directly to the definitions you are looking for, use the menu on the left of the screen to find the definitions you are looking for. *Intervals* is in "Grades 7 to 9" and *frequency table* and *tally* are in "Grades 4 to 6."

TT 3. Open the Excel workbook "Graphing Height of Students" on the Math 8 Multimedia DVD. Enter your frequency data on the first worksheet of the workbook in the cells under "Frequency." You may have to click on the tab at the bottom named "Height of Students" to navigate to this first worksheet.

	A	B
1	**Interval**	**Frequency**
2	110-119	
3	120-129	
4	130-139	
5	140-149	
6	150-159	
7	160-169	
8	170-179	
9	180-189	

Microsoft product screen shot(s) reprinted with permission from Microsoft Corporation.

My Guide

You may be surprised that the graphs change as you enter your data. Only after you have entered all your data will the graphs be correct.

TT 4. Compare the graphs according to how much and what kind of information you can read from each graph. Give an advantage or disadvantage related to each graph type.

TT 5. Separate your data to show the number of boys and girls in each category. Use a table like the following to make a tally.

Heights of Boys and Girls		
Interval	Tally (Boys)	Tally (Girls)
110 to 119		
120 to 129		
130 to 139		
140 to 149		
150 to 159		
160 to 169		
170 to 179		

TT 6. Open the Excel workbook "Graphing Height of Students" on the Math 8 Multimedia DVD. Enter your frequency data on the second worksheet of the workbook in the cells under "Frequency (Boys)" and "Frequency (Girls)." To navigate to the second worksheet, click on the tab named "Height of Boys and Girls."

TT 7. Give an advantage or a disadvantage of each graph on the second sheet of the Excel workbook Graphing Height of Students.

TT 8. Describe the heights of Grade 8 students according to the data you analyzed.

a. Do students have close to the same height, or do the heights vary?
b. In which height interval do most students fit?
c. In which height interval do most girls fit?
d. In which height interval do most boys fit?

Discuss and Share

"Literacy Link" on page 7 of the textbook shows several types of graphs that you can use to represent data. Discuss the following questions with others you know from the discussion board:

- What would be the best graph type to show the populations of the three Prairie provinces?
- What would be the best graph type to show how the populations of the provinces and territories compare to each other and to the whole population of Canada?

- What type of graph would you use to show the country-wide revenue of the latest movie over several weeks after its release?

- What type of graph would you use to show the country-wide revenue of two movies over several weeks after their release?

You may use instant messaging for your discussion.

TT 9. Complete "Explore the Math" question 9 on page 7. Apply what you learned from the discussion about the graphing of populations and movie revenues to complete this question.

Place a copy of the final version of your answers to TT 1 to TT 9 in your Math 8 course folder.

Connect

Now it is time to build on what you discovered during the Explore section.

Read

In "Did You Know?" on page 8 of your textbook, you can read that pizza toppings vary across the world's cultures and countries. "Example 1: Compare Two Graphs" on the same page demonstrates something found out about pizzas in just one location over one weekend—people order pizzas in several ways.

As you read "Example 1" in your textbook, compare how the circle graph and the pictograph each show sales data. Challenge yourself to answer the questions in "Example 1" before you look at the solutions provided after the questions.

Self-Check

SC 1. Complete "Show You Know" at the bottom of page 9 of your textbook to compare how a pictograph and a bar graph represent the same information.

Compare your answer in the Appendix.

Read

"Example 2: Representing Data" on pages 10 and 11 of your textbook shows statistics for two of the top hockey players on the Edmonton Oilers during the 2005–06 season—centres Shawn Horcoff and Jarret Stoll. Examine the data tables to see how many points each player scored during the regular season.

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My Guide

Q: What are some advantages of making a bar graph on grid paper rather than on blank paper?

A: On grid paper, it is easy to make the axes perpendicular to each other. As well, the tick marks on an axis can be placed at regular intervals easily. Also, the grid lines of the grid paper can be used as a guide when drawing the heights of the bars.

Q: How do you determine the maximum value to place along the vertical axis of a bar graph?

A: Look at the greatest value in the set of numbers that is to be represented by the height of bars of the graph. Pick a convenient value above the greatest value. In "Example 2," the greatest value in the set of numbers for the height of the bars is 73. A convenient value above 73 is 80. You might have picked 100, but then there would be extra unused space on the grid above the bars.

Before reading the solution beginning on the bottom of page 10, think about and complete what is being asked in "Example 2." This is a way to check if you fully understand the ideas of this lesson.

Self-Check

SC 2. Complete "Show You Know" at the bottom of page 11 in your textbook. You may want to use "Percent Circles," "0.5 Centimetre Grid Paper," or "1-cm Grid Paper" from the Math 8 Multimedia DVD to create the graphs.

My Guide

Q: How do you use a percent circle to construct a circle graph?

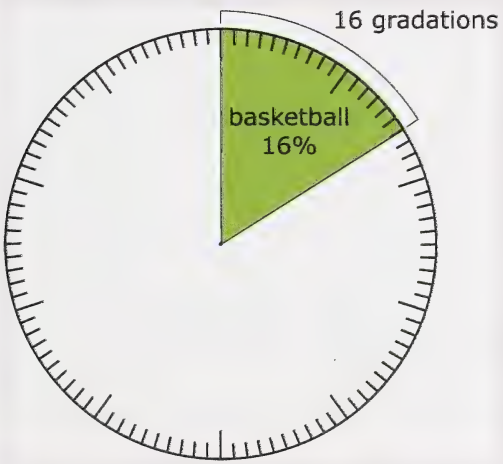
A: The percent circle is like a circular protractor, but instead of being marked out in 360 degrees, it is marked out in 100 gradations. This saves you having to work in degrees. Just calculate the frequency for category as a percent of the total. Then this percentage corresponds to the number of gradations the category should span on the percent circle.

Q: So, how does this work, say, for the category "Basketball"?

A: The frequency for this category is 24.

$$\frac{24}{150} \times 100\% = 16\%$$

So, the sector of the circle graph for category basketball is this.



Compare your answers in the Appendix.

Read

Do you remember the important information you learned from this lesson so far? Read "Key Ideas" on page 12 of your textbook for a short summary of the lesson's main ideas—data can be represented by various types of graphs, and each type of graph has its strengths.

Make notes about this summary for your future study purposes. You might want to add these notes to a foldable like the new one shown on page 4 of your textbook.

Try This

TT 10. Complete "Communicate the Ideas" questions 1, 2, and 3 on page 13 of your textbook. Use the double-bubble organizer "Compare a Bar Graph and a Pictograph" on the Math 8 Multimedia DVD to compare the two types of graphs when answering question 3.

Discuss and Share

Compare your answers with another student on your discussion board. How are your responses different from your online partner? How did you and your partner decide what the correct solution was for the questions? Revise your answers, as needed, as a result of this discussion to reflect an accurate solution to the questions.

Place a copy of your TT 10 answers in your Math 8 course folder.

Self-Check

SC 3. Complete "Check Your Understanding" questions 4 and 6 on pages 13 and 14 of your textbook.

Compare your answers in the Appendix.

SC 4. Complete "Check Your Understanding" questions 7 and 8 on pages 14 and 15 of your textbook.

My Guide

Q: For question 7, how do you know which axis to use for age and height?

A: The data is given in a table in which each column represents a variable. For such data tables, the horizontal axis is generally used for the variable of the left-hand column and the vertical axis is used for the variable of the right-hand column.

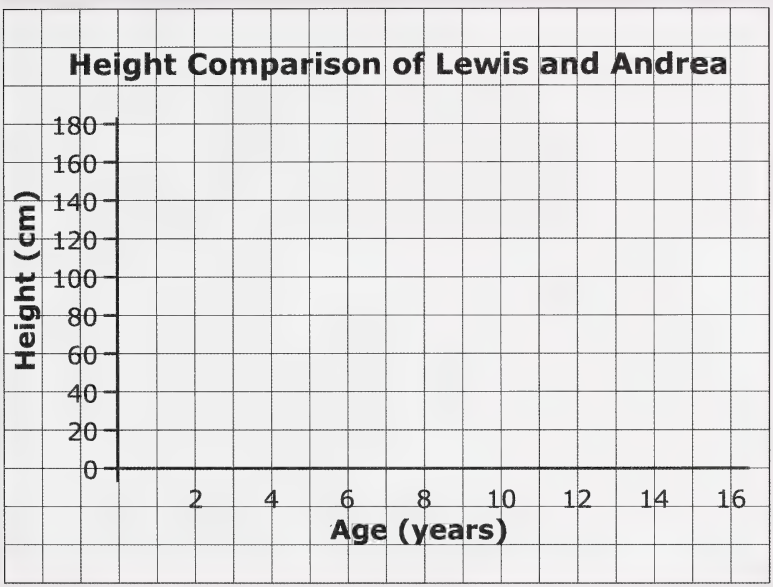
Another way to decide which axis to use is this: Age is one way of describing time. Because time is usually placed along the horizontal axis, that is the axis to use for age. That leaves height for the vertical axis.

Q: What scale should you use on the axis for height? What scale should you use for the axis for age?

A: The greatest height in the data tables is 175 cm. If your scale goes from 0 to 180 cm, you can graph all of the data. If you are using 0.5-cm grid paper, you let each 0.5 cm represent 20 cm.

For the age axis, your scale must include values up to 16. Space the ages so that there is room to show double bars and to keep data points apart.

So your labelled axes could look like this:



Compare your answers in the Appendix.

SC 5. Complete "Check Your Understanding" questions 12 and 14 on page 16 of your textbook.

My Guide

Q: What intervals should be used in the frequency table for question 12?

A: There are various ways to set up the intervals. The data ranges from 40 to 98. The data could be placed in six intervals of ten points. So your frequency table would look like this:

Math Test Scores	
Range	Frequency
40 to 49	
50 to 59	
60 to 69	
70 to 79	
80 to 89	
90 to 99	
Total	

Compare your answers in the Appendix.

Extra Practice

Are you still feeling challenged by the ideas presented in the lesson? More practice might help you become comfortable with the ideas.

Turn to pages 14 to 17 of the textbook. For extra practice, you may complete additional questions from "Check Your Understanding." Then, compare your work with the answers given on pages 476 of the textbook.

Assignment

Go to the Unit 7 Assignment Booklet, and complete "Unit 7: Lesson 1 Question Set."

Going Beyond

The next activity will have you thinking about how the questions you use in a survey can affect the kind of information you collect. To get a better idea of representing survey information, use your creativity to prepare and report on a survey of your own choosing.

"Check Your Understanding" question 17 on page 17 of the textbook will lead you through a survey activity.

Completing this activity will help you to think about the ideas in the next lesson in more depth. Plus, finding out what you want to know from the people around you is always an opportunity for interesting results!

Compare your answers in the Appendix.

Lesson Summary

In this lesson you presented data using different types of graphs and identified advantages of using different types of graph to represent information.

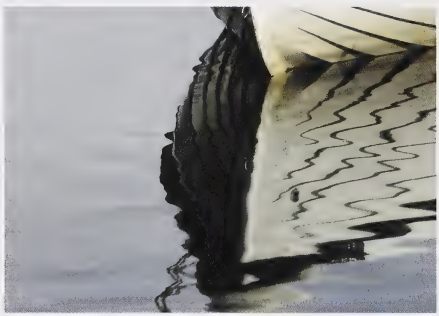
So, how do you choose the type of graph to use when representing data visually? You use the following criteria to guide your choice:

- Circle graphs are best when comparing categories as part of the whole using percent value.
- Line graphs are best when displaying data changing over time.
- Pictographs are best for representing data that can be easily counted and represented using repeated symbols.
- Bar graphs are best for comparing data across one set of categories, and double bar graphs are best for comparing two sets of data across categories

Unit 7: Data Analysis and Probability

Lesson 2: Misrepresenting Data

Get Focused



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Do you enjoy a summer's day at the lake? Then you've likely seen that reflections from even small ripples in the water's surface make objects look distorted.

An amusement park may have a fun house—a place designed to amuse you—with mirrors that are curved to purposely distort images. Seeing yourself or your friends in such mirrors can be very funny.

Turn to page 18 in your textbook *MathLinks 8*. You can read about Kevin who likes to stand in front of a curved mirror in a fun house.

Not all distortions are amusing. As "Literacy Link" on page 18 tells you, changing the appearance of something can mislead people. When graphs are distorted, they misrepresent data and may lead people to make false conclusions.

In the last lesson you discovered that some graphs represent different types of data better than others. In this lesson you will develop skills to recognize graphs that have been distorted and misrepresent data.

This lesson will help you answer the following critical question: How can the format of a graph cause a reader to draw an inaccurate conclusion?

For this lesson, you may need the following, which are available on the Math 8 Multimedia DVD:

- "0.5 Centimetre Grid Paper"
- "1-cm Grid Paper"

Assignments

Your assignment will consist of the following:

- posting to your discussion board
- adding to your Math 8 folder
- completing Unit 7: Lesson 2 Questions Set

Explore

In the "Explore the Math" activity presented on pages 18 and 19 of the textbook, you will look at different graphs that display the same data. This will help you to answer the following question: What are some ways to misrepresent data?

Work with a partner as you complete the "Explore the Math" activity. Your partner may be someone assigned by your teacher, a friend or a family member, or someone from your discussion board.

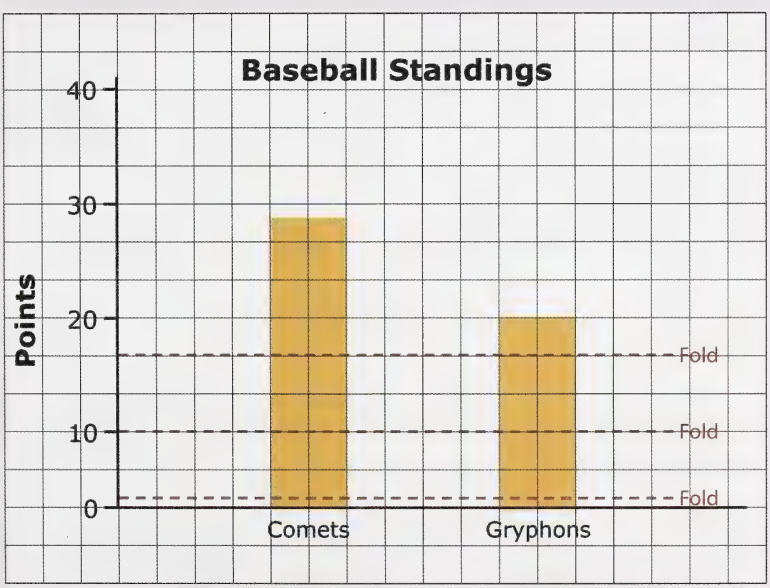
With your learning partner, discuss the following:

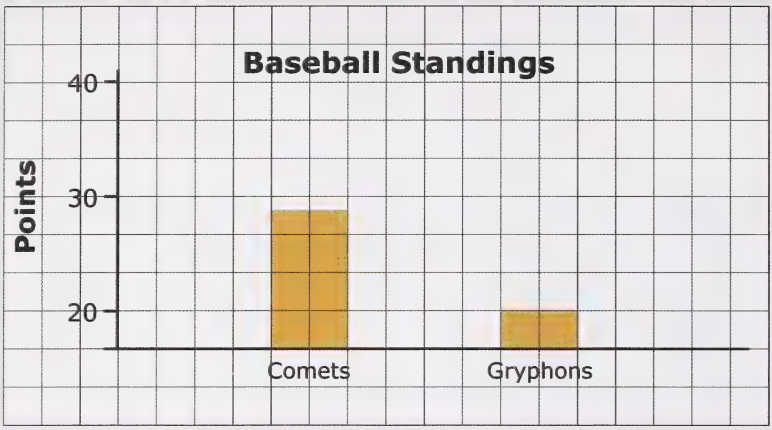
- How do the graphs on the bottom of page 18 for the sales differ?
- What is the meaning of the little squiggle at the bottom of the vertical axis, just above the origin, in Graph B?

My Guide

Q: How can a break in the vertical axis change a graph?

A: The break in the vertical axis brings the points of the graph closer to the horizontal axis. You can see this effect by making a graph and then folding away a horizontal strip just above the horizontal axis. This can be done by folding along three horizontal lines. Here's an example of what happens when a graph is modified this way.





Notice that after the folds, the difference in scores makes a bigger impression.

Q: Imagine that a representative from a phone company was showing you how much money you'd save if you purchased a cellphone plan from the phone company. Why is it important for you to notice the scale on the vertical axis?

A: If you are reading information off a graph and you do not notice where the vertical axis starts, you might think that one phone plan is less expensive than another plan. In reality, maybe the cellphone plan is less expensive by only a few cents and has fewer features.

So, if you can see exactly where the axis starts and what the scale is, you will have a better idea about the information that is shown by the graph.

Self-Check

SC 1. Complete "Explore the Math" questions 1, 2, 3, and 4 on page 19. Remember to work with a learning partner.

Compare your answers in the Appendix.

Your study of the two different graphs for the same data may make you more aware of how people draw conclusions from graphs. In the next question you can describe how conclusions may be affected by the scale of a graph.

Try This

TT 1. Complete "Explore the Math" question 5 on page 19. Continue to work with a learning partner.

Place a copy of the final version of your answer to TT 1 in your Math 8 course folder.

Connect

Let's now build on what you and your partner discovered when you completed the Explore section.

Read

Read "Example 1: Distorting the Scale" on pages 19 and 20 of the textbook to see how a graph can exaggerate a student's progress in writing tests. As you work through the example, cover the solutions with a piece of paper and try to answer the questions. Then uncover the solution as you answer each question.

Self-Check

SC 2. Complete "Show You Know" in the middle of page 20 of the textbook to see how changes in gasoline prices can be "blown out of proportion."

Compare your answers in the Appendix.

Read

"Example 2: Distorting the Visuals" on pages 20 and 21 of the textbook shows a pictograph of favourite pets for one group of Grade 8 students. The pictograph has no axes to break, yet it was made in such a way that it is misleading. Look at the pictograph to see how it could trick you.

Self-Check

SC 3. Complete "Show You Know" in the middle of page 21 of the textbook.

Compare your answers in the Appendix.

Read

You are familiar with the idea that you have to check for gaps in the vertical axis of a bar graph to avoid being misled by the information in the graph. "Example 3: Distorting the Size of Bars" on pages 21 and 22 of your textbook shows still another way a bar graph can be misleading.

Self-Check

SC 4. Complete "Show You Know" in the middle of page 22 of your textbook.

Compare your answers in the Appendix.

Read

Read "Key Ideas" on page 22 of the textbook for a short summary of the lesson's main idea—graphs can be mislead people into making false conclusions.

Discuss and Share

Post to your discussion board the ways you discovered graphs can be distorted. Also, give an example of a situation where a distortion should be used. Respond to other students. Explain whether or not you agree with the examples that other students provided.

Make notes to use as a study tool from "Key Ideas" and from the discussion board. You might want to add these notes and ideas to a foldable like the new one shown on page 4 of the textbook.

Try This

TT 2. Complete "Communicate the Ideas" questions 1 and 2 on page 23 of your textbook.

Place a copy of your answers in your Math 8 course folder.

Self-Check

SC 5. Complete "Check Your Understanding" questions 5, 6, 8, and 9 on pages 24 and 25 of your textbook.

SC 6. Complete "Check Your Understanding" questions 10, 11, 12, 13, 15, 16, and 18 on pages 25 and 26 of your textbook.

Compare your answers in the Appendix.

Extra Practice

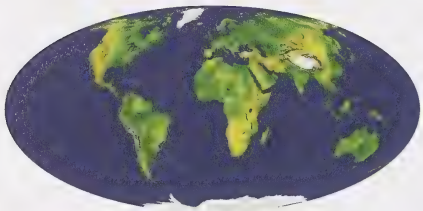
Are you still feeling challenged by the ideas presented in the lesson? More practice might help you become comfortable with the ideas.

Turn to pages 23 to 27 in the textbook. For extra practice, you may complete additional questions from "Check Your Understanding." Then, compare your work with the answers given on pages 478 of the textbook. Be sure to talk to your teacher if there are any questions that puzzle you.

Assignment

Go to the Unit 7 Assignment Booklet, and complete "Unit 7: Lesson 2 Question Set."

Going Beyond



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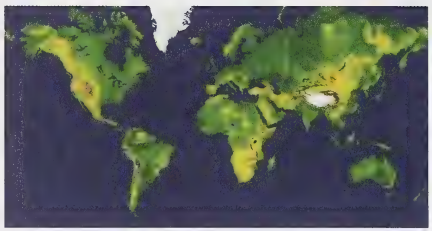
Mollweide world map



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Mercator world map

Even when graphs can provide a very realistic representation, they are still occasionally designed to give a distorted view. However, distortion is unavoidable when representing the spherical surface of Earth as a world map.

The Mollweide and Mercator maps both distort the surface of the world but in different ways. Identify differences and similarities between the Mollweide and Mercator world maps.

You may find it helpful to talk with someone else on your discussion board about world maps and share your questions and ideas relating to the Mollweide and Mercator world maps.

Compare your answers in the Appendix.

Lesson Summary

In this lesson you investigated how the format of bar graphs, pictographs, line graphs, and circle graphs can be distorted so that they misrepresent data.

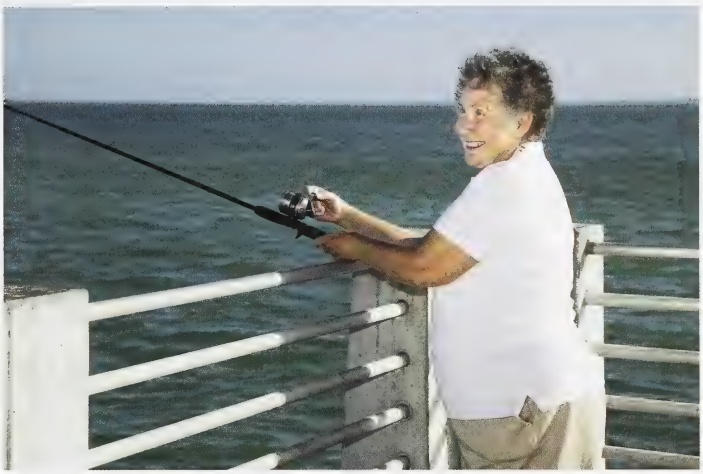
The format of a graph can cause a reader to draw a false conclusion. This happens when the scale of one of the axes is changed, when a break is put into one of the axes, or when the size of visuals is not the same.

In the next lesson you will critique graphs for their effectiveness in representing data.

Unit 7: Data Analysis and Probability

Lesson 3: Critiquing Data Presentation

Get Focused



© Lisa F. Young/Dreamstime

About one Canadian in five enjoys fishing as a pastime. You or someone in your family may be among the many who fish as a sport.

Besides sport fishers, there are also many commercial fishers. These are fishers who make an income from fishing. To make sure that fish populations will be available for future generations, the number of fish that may be harvested is controlled by government fishery officials through quotas. Quotas place upper limits on the numbers of fish that may be caught during a fishing season.

Turn to page 28 in your *MathLinks 8* textbook to read about an improvement in the management of fish stocks. DNA testing was used to distinguish between stronger and weaker stocks. When fisheries' officials placed higher quotas on stronger fish stocks while protecting weaker stocks, fishers started catching more Chinook salmon, according to the story.

Does the bar graph support the story? Answering this question involves judging (i.e., critiquing) the graph for how well it presents data. You may think about this question now, but you will revisit the question in the Explore part of this lesson.

In this lesson you will critique the way graphs—circle graphs, line graphs, bar graphs, and pictographs—are used to represent data.

As in previous lessons, you will need materials such as grid paper, a ruler, and percent circles for making graphs.

This lesson will help you answer the following critical question: What factors should you consider when judging whether a graph represents information well?

Assignments

Your assignment will consist of the following:

- posting to your discussion board
- adding to your Math 8 folder
- completing Unit 7: Lesson 3 Question Set

Explore

In the "Explore the Math" activity presented on pages 28 and 29 of the textbook, you are to critique the graph "DNA Testing Increases Chinook Catch."

Work with a partner as you complete the "Explore the Math" questions. Your partner may be someone from your discussion board. You may use instant messaging to discuss how appropriate the title "DNA Testing Increases Chinook Catch" is for the graph and how appropriate the effect of "stacking" the bars is for the readability of the graph.

My Guide

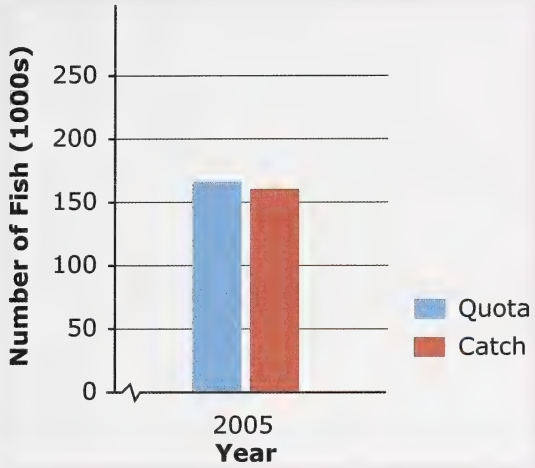
Q: How can I critique or judge the graph on page 28? I am not familiar with the data that the graph is based on.

A: You are not in a position to critique the data without doing a lot of research. But critiquing the data is not the point. You are to judge just the graph for how well it provides a visual representation of the data.

Q: How high should I consider the bar for "Quota" to be? Does it start from the top of the "Catch" bar, or does it start from the horizontal axis?

A: The height of the bar for "Quota" should be measured from the horizontal axis. You can infer that from the bars for 2005. You know that the number of fish caught cannot exceed the quota for the year. If the height of the "Quota" bar was only the part visible above the "Catch" bar, the "Quota" bar would be too short.

The "Quota" and the "Catch" data for 2005 are shown here in a double bar graph for easier interpretation.



Adapted from Bruce McAskill et al., *MathLinks 8: Student Resource* (Toronto: McGraw-Hill Ryerson, 2008), 28. Reproduced by permission.

Q: How did the Chinook salmon catch change from the 2001 season to the 2002 season? What else happened during this time?

A: Just look at the bars for "Catch" on the graph: The catch increased. This was when DNA testing was introduced to identify which fish stocks fishers could focus on.

Self-Check

SC 1. Complete "Explore the Math" questions 1, 2, and 3 on page 28. Remember to work with a learning partner.

Compare your answers in the Appendix.

Discuss and Share

Discuss the following question with other students on your discussion board: What is the one most important factor to consider for critiquing graphs? Indicate to several other students whether or not you agree with the factors they list.

Try This

TT 1. Complete "Explore the Math" question 4 on page 29.

Place a copy of the final version of your answer in your Math 8 course folder.

Connect

Let's now build on what you and your partner discovered during the completion of the Explore section.

Read

Read "Example 1: Critiquing a Graph" on pages 29 and 30 of the textbook to see if you can critique a graph. Remember that critiquing is not just about finding faults in visual representation. You should note good points during a critique.

Self-Check

SC 2. Complete "Show You Know" on the bottom of page 30 of the textbook.

Compare your answers in the Appendix.

Read

Read "Key Ideas" on page 31 of the textbook for a short summary of the lesson's main ideas about critiquing graphs.

Make notes about this summary for your future study purposes. You might want to add these notes to a foldable like the new one shown on page 4 of your textbook.

Try This

TT 2. Complete "Communicate the Ideas" questions 1, 2, and 3 on page 31 of your textbook.

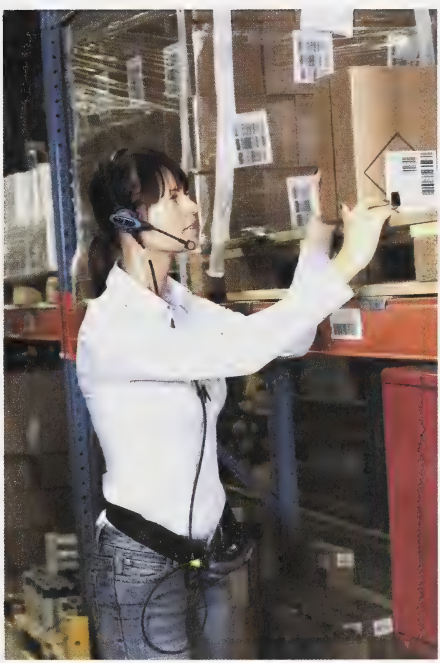
Place a copy of your answers in your Math 8 course folder.

Self-Check

SC 3. Complete "Check Your Understanding" questions 4, 5, and 6 on page 32 of your textbook. Be sure to take the time to thoroughly explain your thinking. This will help you to fully cement the ideas in your mind.

SC 4. Complete "Check Your Understanding" questions 7, 8, and 10 on pages 33 and 34 of your textbook. Be as clear as possible when you explain your thoughts. Your answers can help you to review the main ideas of the lesson as you work.

Compare your answers in the Appendix.



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Extra Practice

If you're still feeling challenged by the ideas presented in the lesson, more practice might help you.

Turn to pages 32 to 35 of the textbook. For extra practice, you may complete additional questions from "Check Your Understanding." Then compare each question you complete with the answers given on pages 480 and 481 of the textbook.

Assignment

Go to the Unit 7 Assignment Booklet, and complete "Unit 7: Lesson 3 Question Set."

Going Beyond

Most employers must pay their employees a minimum wage. The minimum wage has been going up in the province of Manitoba.

Answer "Check Your Understanding" question 11 on page 34 of the textbook to learn more about the recent trend in Manitoba's minimum wage. See if you can figure out the distortions in the graphs that supposedly represent the exact same information.

Compare your answers in the Appendix.

Lesson Summary

In this lesson you critiqued the way graphs were used to represent data.

Can you remember the factors you should consider when you judge whether or not a graph represents information well? You must include these factors when making this judgment:

- Ask if the type of graph—such as a circle graph, a line graph, or a pictograph—is the best choice to represent the data.
- Ask if the format of graph is designed to represent the data accurately. For example, is the scale appropriate? Are there breaks or gaps in the axes?
- Ask if the graph provides relevant information. Does the graph have anything to do with the topic under investigation?

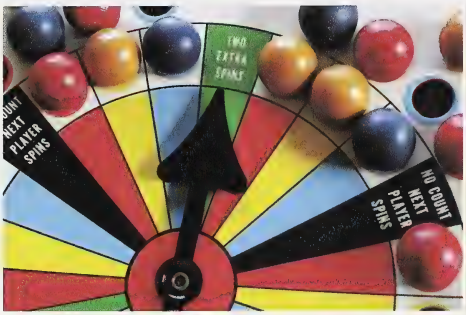
Remember your Unit Problems. Refer to the Getting Started section of the Unit 7 Summary. With the completion of this lesson, you have the background to complete the Unit Problem 1.

In the last three lessons you used graphs to visually represent data describing events that actually occurred in the past. In the next lesson you will investigate the probability or likelihood of future events.

Unit 7: Data Analysis and Probability

Lesson 4: Determining Probabilities Using Tree Diagrams and Tables

Get Focused



left: © Suzanne Tucker/Dreamstime
centre: © Mihails Jershovs/Dreamstime
right: © Sven Larsen/Dreamstime

In many games, a roll of the dice, the selection of a card, or the spin of a spinner is used to decide some aspect of the game, such as how far you move on a board.

Turn to page 410 in your textbook *MathLinks 8* to read about the use of a simple spinner and a four-sided die to decide the characteristics of a test. It may appear that Ms. Pascal was rather blasé about the format and length of the test, as she left this to chance.

The question in the textbook reading asks how Ms. Pascal knows that the spinning and the rolling are **independent events**. Maybe you are not ready to agree with Ms. Pascal. Then, think of two real-world events that are independent—the temperature outside and the day of the week. You'd likely agree that the day of the week does not have any connection to the temperature. For example, Tuesdays are not always hot and Fridays are not always cold.

independent event: an event in which the outcome of one event has no effect on the outcome of another

probability: the likelihood of a given outcome occurring

So, does Owen's spinning of the spinner affect Ava's roll of the dice or vice versa? You'd likely agree that the events have no more effect on each other than the day of the week and the temperature outside do. So, what Owen's spinner lands on and what number Ava's dice land on can be thought of as independent events. Another way to express this independence is this statement: What region the spinner lands on has no effect on the **probability** of the die landing on a given number, and what number the die lands on has no effect on the probability of the spinner landing on a given number.

In this lesson you will determine the probability of two given independent events using special organizers such as tree diagrams and tables.

You may want to use a die and spinner to see what the possible formats of Ms. Pascal's test could be. For a four-sided die, you can use a regular six-sided die and just ignore the throws with the six-sided die that land on 5 or 6.

You can make a three-region spinner by spinning a paper clip as a pointer on a disc divided into three equal regions. In a similar way, you can make a four-region spinner. You may use the templates "Three-region Spinner" and "Four-region Spinner" as found on the Math 8 Multimedia DVD.

This lesson will help you answer the following critical question: How can you determine probabilities using tree diagrams and tables?

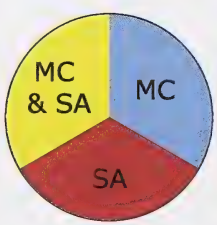
Assignments

Your assignment will consist of the following:

- posting to your discussion board
- adding to your Math 8 folder
- completing Unit 7: Lesson 4 Question Set

Explore

In the "Explore the Math" activity presented on pages 410 and 411 of the textbook, you are to analyze how the simple spinner and the four-sided die can affect the format of the test for Ms. Pascal's class.



Find a partner to help you work through the Explore section. Your partner will help you gather data and work through the questions. Your partner may be someone in your class or a family member. If you are going to experiment with a spinner and die, then you can take turns recording the outcomes while the other person spins the spinner or rolls the die.

© Sven Larsen/Dreamstime

As you begin to record the outcomes, you may discover an efficient way to record the results so that all of the possible outcomes can be predicted. All of the possible outcomes make up the **sample space**.

sample space: the collection of all the possible outcomes of a probability experiment

Self-Check

SC 1. Complete "Explore the Math" questions 1, 2, 3, and 4 on pages 410 and 411.

My Guide

Q: How can the outcomes MC, 20; MC, 5; SA, 5; MC & SA, 15; and MC & SA, 5 be recorded in an organized way?

A: You could use the kind of branching found in concept maps. That will show how these outcomes relate to each other. Here's an example.

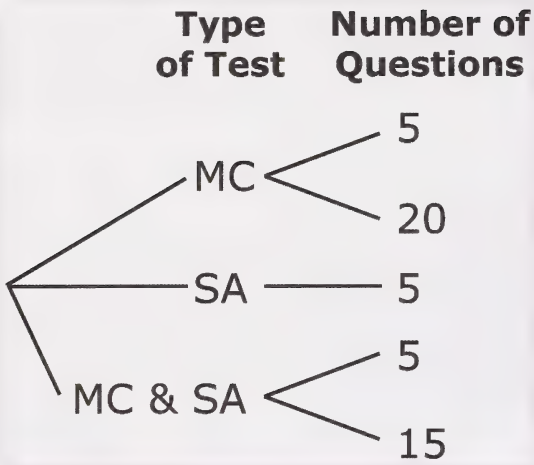
Q: The first level of the diagram has the entries MC, SA, and MC & SA. What aspect of the test do these symbols describe?

A: These symbols indicate the format or type of test.

Q: The second level of the diagram has the entries 5, 15, 20. What aspect of the test do these symbols describe?

A: These symbols indicate the length of the test or the number of questions on the test.

Q: How could the diagram be simplified so that only new information is indicated at each level?



The diagram does not yet have branches for all of the possible outcomes. When you complete this diagram with all the possible numbers of questions for each type of test, you will have a tree diagram for the situation.

Q: Is there a way to organize the outcomes in a rectangular table?

A: Yes. Rather than list the outcomes in one long list, a table with the following headings would allow you to record outcomes and to see which other outcomes are possible.

		Number of Questions			
		5	**10**	**15**	**20**
Type of Test	MC	MC, 5			MC, 20
	SA	SA, 5			
	MC & SA	MC & SA, 5		MC & SA, 15	

Compare your answers in the Appendix.

Try This

TT 1. Complete "Explore the Math" question 5 on page 411.

Discuss and Share

Share your ideas with your partner about using the sample space to determine the probabilities related to various types of tests students could have to write. As you work through question 5, it is important for you both to agree on the solutions for your responses. If your answers are different, discuss where a potential mix up could have happened in your reasoning.

Place a copy of the final version of your answers in your Math 8 course folder.

Connect

Let's now build on what you and your partner discovered during the Explore.

Read

Look at "Did You Know?" in the margin of page 411. Can you see why a four-sided die must be read differently from six-sided one? A four-sided die is useful in simulating a probability experiment that is based on an event having four equally likely outcomes.

favourable outcome: a successful outcome in a probability experiment

A favourable outcome is any outcome for which you are trying to find out a probability.

Read "Example 1: Determine Probabilities From a Tree Diagram" on pages 411 and 412 of your textbook to see how you can make a tree diagram for a probability experiment composed of two spins of a spinner. Do you see why some of the outcomes in the sample space are considered to be **favourable outcomes**?

Self-Check

SC 2. Complete "Show You Know" on page 413 of your textbook.

Compare your answers in the Appendix.

Read

Read "Example 2: Determine Probabilities From a Table" on pages 413 to 415 of your textbook to see how you analyze a probability experiment composed of the roll of two six-sided dice. Can you see why a table is more suitable than a tree diagram for representing the outcomes?

Self-Check

SC 3. Complete "Show You Know" on page 415 of your textbook section.

Compare your answers in the Appendix.

Read

A main idea of this lesson can be expressed by this mathematical statement:

$$\text{probability} = \frac{\text{number of favourable outcomes}}{\text{number of possible outcomes}}$$

Do you also remember the additional important information you learned from this lesson so far? Read "Key Ideas" on page 415 of your textbook for a complete listing of the main ideas in this lesson.

Make notes about this summary. It may be time for you to make a new foldable. You might want to add these notes to a foldable like the new one shown on page 408 of your textbook.

Try This

TT 2. Complete question 1 of "Communicate the Ideas" on page 416 of your textbook.

Place a copy of your answers in your Math 8 course folder.

Self-Check

SC 4. Complete "Check Your Understanding" questions 3 and 5 on pages 416 and 417 of your textbook.

SC 5. Complete "Check Your Understanding" questions 7, 8, 9, and 10 on page 417 of your textbook.

Compare your answers in the Appendix.

Extra Practice

For extra practice, you may wish to complete additional questions from "Check Your Understanding." Turn to pages 416 and 417 of the textbook. Then compare your work with the answers on pages 509 to 512 of the textbook. The instant feedback from checking your responses frequently can help you to gauge the progress you are making.

Assignment

Go to the Unit 7 Assignment Booklet, and complete "Unit 7: Lesson 4 Question Set."

Going Beyond

In this lesson you have been developing effective strategies to determine the probabilities of two independent events. Can you extend these strategies to determine the probabilities of three events? See if you are up to the challenge by answering the following questions from your textbook.

- Complete question 2 of "Communicate the Ideas" on page 416.
- Complete question 11 of "Check Your Understanding" on page 418.

Compare your answers in the Appendix.

Lesson Summary

In this lesson you determined the probability of two given independent events using tree diagrams and tables as organizers of outcomes. With a count of the favourable outcomes and the total number of outcomes in one of these organizers, the following mathematical statement can be used to calculate the probability:

$$\text{probability} = \frac{\text{number of favourable outcomes}}{\text{number of possible outcomes}}$$

Remember to work on your unit problems. Refer to the Getting Started section of the Unit 7 Summary to work on the Unit Problem 2.

In the next lesson you will turn your attention to the number of possible outcomes for a given probability experiment.

Unit 7: Data Analysis and Probability

Lesson 5: Outcomes of Independent Events

Get Focused

The school sports day is a fun day of the year for many students. The day can be enjoyed in so many ways—the competition, the cheering, the team spirit, and just being outside with classmates.

When participating in a lot of physical activity, as on a school sports day, a student can build a hearty appetite. That's when a lunch special from the school cafeteria can be a welcome treat.

The lunch special in their school's cafeteria will be enjoyed by students Ethan and Sarah. Turn to page 419 in your textbook *MathLinks 8* to find why Sarah may need to take a little more time than Ethan to order her meal.

The three choices that Sarah makes in ordering her lunch will result in a made-to-order lunch. Using the terms of the previous lesson, you can think of her choices as *events*, as in a probability experiment. Then the made-to-order lunch can be considered to be an *outcome* of the events.

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In this lesson you will determine the number of outcomes of independent events.

This lesson will help you answer the following critical question: How can you determine the number of outcomes for a probability experiment?

Assignments

Your assignment will consist of the following:

- posting to your discussion board
- adding to your Math 8 folder
- completing Unit 7: Lesson 5 Question Set

Explore

In the previous lesson you used the following formula.

$$\text{probability} = \frac{\text{number of favourable outcomes}}{\text{number of possible outcomes}}$$

This formula shows that probability calculations are based on the total number of possible outcomes. In the "Explore the Math" activity presented on pages 419 and 420 of the textbook, you will develop and apply methods of finding this important number.

It is important to work with a partner to compare methods of finding the number of possible outcomes, as you complete the "Explore the Math" questions. This discussion is important to help you learn a concept. Your partner may be a classmate, a friend, or someone from your discussion board. Contact your teacher if you are having difficulties finding a partner.

Self-Check

SC 1. Complete "Explore the Math" questions 1 and 3 on page 419.

My Guide

Q: Could I use a table—as I did in the previous lesson—to show the possible outcomes of Sarah's choices?

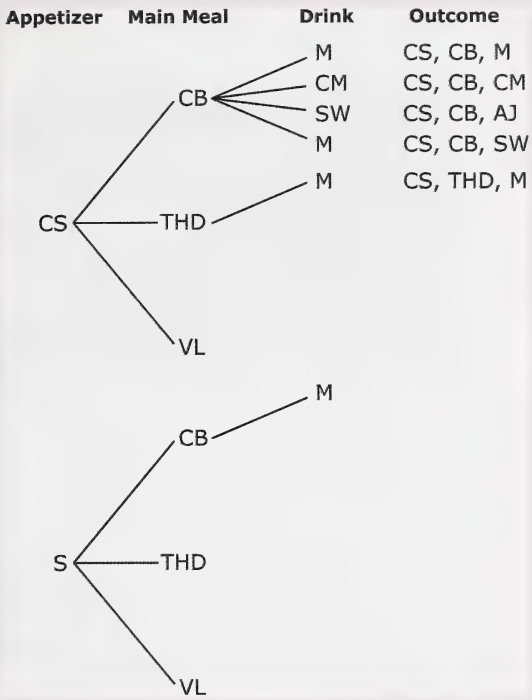
A: You could try to build such a table.

		Appetizers	
		Chicken Soup	Salad
Main Meal	Cheeseburger		
	Turkey Hot Dog		
	Vegetable Lasagna		

But then, where would you put the choices for drinks? There's no place for drinks on the table.

Q: In the previous lesson, the probability experiments had only two events. We used tree diagrams to show outcomes for those experiments. Would that work when there are three events?

A: Yes, a tree diagram would work. Just add an extra layer. It would look like this:



Note: This is just a partial sample of the tree. Completing this tree diagram would yield all the outcomes.

Compare your answers in the Appendix.

SC 2. Complete "Explore the Math" questions 4 and 5 on page 420. Remember to work with a partner as you complete these questions. Be sure to check out "Literacy Link" on page 420 to see why a given outcome can be written in more than one way.

Compare your answers in the Appendix.

Try This

TT 1. Complete "Explore the Math" question 6 on page 420. Continue to work with a learning partner. Remember that you and your partner are trying to develop a method together. It's important to discuss the merits of each others' ideas as you work. Through your discussions, develop a method of calculating the total number of outcomes *without* using a tree diagram or a table.

Place a copy of the final version of your answers in your Math 8 course folder.

Connect

Build on what you and your partner discovered during the Explore section.

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Read

Read "Example 1: Determine the Total Number of Outcomes From Two Events" on page 420 of the textbook. See which method of finding the number of outcomes is more convenient.

Self-Check

SC 3. Complete "Show You Know" in the middle of page 421 to determine how many ways there are to order a sandwich. Be sure to carry out the check.

Compare your answers in the Appendix.

Read

"Example 2: Determine the Total Number of Outcomes From Three or More Events" on pages 421 and 422 of the textbook shows how to find the number of outcomes resulting from more than two events. Can you see why the tree diagram has three levels and the multiplication has three factors?

Self-Check

SC 4. Complete "Show You Know" on page 422 of your textbook. Ensure you check your responses using an alternate method.

Compare your answers in the Appendix.

Read

Read "Key Ideas" on pages 422 and 423 of your textbook for a brief summary of the lesson's main idea—the total number of outcomes can be determined using a tree diagram, a table, and multiplication.

Make notes about this summary to use for future study purposes. You might want to add these notes to a foldable like the one shown on page 408 of your textbook.

Try This

TT 2. Complete "Communicate the Ideas" questions 1 and 2 on page 423 of your textbook. Check with your teacher if you are feeling a bit stumped or need some help getting started with these questions.

Place a copy of your answers in your Math 8 course folder.

Self-Check

SC 5. Complete "Check Your Understanding" questions 3, 5, and 6 on pages 423 and 424.

SC 6. Complete "Check Your Understanding" questions 7, 8, 10, and 11 on page 424.

Compare your answers in the Appendix.

Extra Practice

If you are still feeling challenged by the ideas presented in the lesson, extra practice will help you become more comfortable with the lesson ideas.

Turn to pages 423 and 424 of the textbook. For extra practice, you may complete additional questions from "Check Your Understanding." Then compare your work as you finish each question with the answers on pages 510 and 511 of the textbook. Your teacher is also available to help you with questions you may have.

Assignment

Go to the Unit 7 Assignment Booklet, and complete "Unit 7: Lesson 5 Question Set."

Going Beyond

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There is a variety of license plates seen when travelling in Alberta—especially during the summer when tourists visit with cars from other Canadian provinces or from the United States.

Did you know that the design of Alberta's license plates is almost 25 years old? That has led to a problem. License-plate numbers based on this old design are running out. How can you determine how many numbers are available for a particular sequence of characters in a design?

Answer "Check Your Understanding" question 16 on page 425 of the textbook.

Compare your answers in the Appendix.

Lesson Summary

The critical question posed at the beginning of this lesson was the following: How can you determine the number of outcomes for a probability experiment? This lesson showed the answer to this question.

You can determine the number of outcomes by using the following:

- tree diagrams
- tables
- operation of multiplication (The numbers of possible outcomes for the individual event are multiplied together.)

In the next lesson you will solve probability problems with calculations using fractions.

Unit 7: Data Analysis and Probability

Lesson 6: Determining Probabilities Using Fractions

Get Focused

Imagine a game where the prizes were all-expense excursions that took winners dancing at a major First Nations event, scuba diving on a coral reef, or riding an elephant.

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Turn to page 426 of the textbook *MathLinks 8*. Read about an imaginary TV program called "Wheel of Thrills" that offers contestants a variety of interesting excursions.

You may already have a strategy to determine the probability that Erv or Al will win an excursion that involves swimming. But in this lesson you will develop and apply a new strategy based on fractions to solve such probability problems.

In this lesson you will generalize and apply a rule for determining the probability of independent events.

This lesson will help you answer the following critical question: How can you determine the probability of independent events using a rule?

You need red and yellow pencil crayons or marking pens to complete this lesson.

Assignments

Your assignment will consist of the following:

- posting to your discussion board
- adding to your Math 8 folder
- completing Unit 7: Lesson 6 Question Set

Explore

In the "Explore the Math" activity presented on pages 426 and 427 of the textbook, you will use fractions to determine probabilities for contestants playing the game "Wheel of Thrills."

You may use the "Table of Outcomes" that is shown on the bottom of page 426. You can find "Table of Outcomes" on the Math 8 Multimedia DVD. Either save the chart to your computer or print a copy from the Math 8 Multimedia DVD.

Self-Check

SC 1. With a learning partner, complete "Explore the Math" questions 1 and 2 on pages 426 and 427.

My Guide

Q: Which row or rows should be shaded in—the spinner landing on Al's name or the spinner landing on Erv's name?

A: Think of the successful outcomes of the spinning event. Landing on Al's name is a successful outcome, and landing on Erv's name is a successful outcome. So both rows having names should be shaded in. This question points out the special way you have to treat the word *or* in probability problems.

Q: Which column or columns should be shaded in—swimming with dolphins or scuba diving on a coral reef?

A: Answering this question again relates the interpretation of the word *or*. Think of the successful outcomes of the rolling of the die. Landing on 3 is a successful outcome and so is landing on 5. These two columns should both be shaded in.

Compare your answers in the Appendix.

Discuss and Share

The "Explore the Math" activity gave you specific steps to follow for colouring a table of outcomes. The way of colouring a table of outcomes is similar to the folding of paper to model the multiplication of fractions.

Do you remember how you modelled the multiplication of fractions by folding paper? With your partner, you may want to look back to textbook page 211 to refresh your memory.

Compare the colouring of the table of outcomes to the folding of paper to model the multiplication of fractions. This comparison will help you see how multiplication can be used to calculate probabilities.

Try This

It is time to make some conclusions about the use of multiplication and fractions in solving probability problems involving two independent events. Remember to work with a learning partner as you answer these questions.

TT 1. Compare the colouring of the table of outcomes to the folding of paper to model the multiplication of fractions.

TT 2. Complete question 3 of "Explore the Math" on page 427 of the textbook. **Note:** The two references in question 3 should be to question 2.b), not question 2.c).

Place a copy of the final version of your answers in your Math 8 course folder.

Connect

Let's now build on what you and your partner discovered about probability when you completed the activity in the Explore section.

Read

Read "Example 1: Calculating Probabilities Using a Table and Multiplication" on pages 427, 428, and 429 of the textbook to see how the probability for an experiment can be calculated using a table and the operation of multiplication.

Recall the meaning of this type of notation: $P(A)$ represents the probability of outcome A actually occurring during a probability experiment.

Read "Example 2: Calculating Probabilities Using a Tree Diagram and Multiplication" on pages 429, 430, and 431 of the textbook. This example, like the first example, shows the calculation of the probability of two independent events. "Example 2," however, shows the use of a tree diagram instead of a chart. As you go over the two examples, it is important to look at how the probability of individual events can be used to calculate the combined probability.

Self-Check

SC 2. Complete "Show You Know" at the bottom of page 429 to practise calculating combined probability using the probabilities of individual events.

Compare your answers in the Appendix.

Read

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A fair coin is one that does not tend to land on any one side more than the other.

Think of a probability experiment in which two fair coins—a nickel and a penny, for example—are flipped. Then the probability of getting two heads can be calculated. For the flip of either coin, $P(\text{H}) = \frac{1}{2}$.

$$P(\text{H, H}) = \frac{1}{2} \times \frac{1}{2}$$
$$= \frac{1}{4}$$

The probability of two heads showing after the coins are flipped is $\frac{1}{4}$. This calculated probability is a **theoretical probability**.

On the other hand, you could run 100 trials in which you flipped both coins. Then you could compare the number of times both coins came up heads. If two heads appeared 24 times, you could conclude that the probability of two heads is $\frac{24}{100}$. This fraction is an **experimental probability** found directly through experimentation and observation.

theoretical probability: a probability value based on calculation

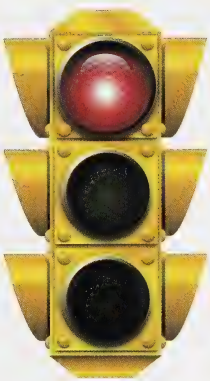
The following is one calculation that gives a theoretical probability for a given favourable outcome.

$$\text{probability} = \frac{\text{number of favourable outcomes}}{\text{number of possible outcomes}}$$

experimental probability: a probability value based on experimental results

simulation: an experiment designed to model a real situation

A probability experiment may be used as a simulation.



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Go over "Example 3: Simulations" on pages 430 and 431 of the textbook to compare the two types of probability—theoretical and practical—to find out the likelihood that Gina will have two red lights on the way to her soccer game. Also pay close attention to how a **simulation** can be used to substitute for trial drives to the soccer game.

Self-Check

SC 3. Complete "Show You Know" near the top of page 431.

Compare your answers in the Appendix.

Read

This lesson involved a lot of new terms. Review "Key Ideas" on page 431 of the textbook for a short summary of the lesson's main ideas: Probabilities can be found by multiplying probabilities of individual events, and simulations provide experimental probabilities.

Make notes about this summary for your use for future study purposes. You might want to add these notes to a foldable like the new one shown on page 408 of the textbook.

Try This

TT 3. Complete "Communicate the Ideas" questions 1 and 2 on page 432.

Place a copy of your completed answers in your Math 8 course folder.

Self-Check

SC 4. Complete "Check Your Understanding" questions 4, 6, 8, and 9 on pages 432 and 433.

SC 5. Complete "Check Your Understanding" questions 10, 11, 12, and 13 on pages 433 and 434.

Compare your answers in the Appendix.

Extra Practice

Are you still feeling challenged by the ideas presented in the lesson? More practice might help you become comfortable calculating the probability of events.

Turn to pages 432 to 434 of the textbook. For extra practice, you may complete additional questions from "Check Your Understanding." Then compare your work with the answers on pages 511 and 512. Be sure to check your answers as you complete each question. This will help to keep you on track and let you know when you have developed a better understanding of the ideas of this lesson. Your teacher is also available to help you.

Assignment

Go to the Unit 7 Assignment Booklet, and complete "Unit 7: Lesson 6 Question Set."

Going Beyond

You've determined the probability of an experiment from the probability of its individual events. Now see if you can work backwards to determine the probability of a single event of an experiment.

Complete "Check Your Understanding" question 17 on page 435 of the textbook. Bear in mind that for any two independent events A and B, $P(A, B) = P(A) \times P(B)$.

Compare your answers in the Appendix.

Lesson Summary

In this lesson you generalized and applied a rule for determining the theoretical probability of independent events and used simulations to determine experimental results.

You started this lesson with this question about a probability experiment involving more than one event: How can you determine the probability of independent events using a rule?

To find the probability of independent events, you multiply the probabilities of the individual events of the experiment.

This rule can be expressed as follows:

$P(A, B) = P(A) \times P(B)$

$P(A, B)$ is the probability of independent events A and B occurring together, $P(A)$ is the probability of event A occurring individually, and $P(B)$is the probability of event B occurring individually.

Congratulations on working to get to this point in the course—this is the last lesson in Math 8! After this lesson comes the Unit Summary.

Unit 7: Data Analysis and Probability

Unit 7 Summary

Getting Started

Many pastimes involve data or some form of chance, where outcomes are unpredictable. If you follow the music scene, you need to be able to read data tables and graphs. In games involving chance, being able to work out the probability of outcomes in the sample space helps you play the game as well as possible.

The Unit Problem consists of two parts:

- Unit Problem 1: Notes on Music
- Unit Problem 2: A Fair Chance

For Unit 7, you will explore the use of data and chance as you complete two Unit Problems.

Unit Problem 1: Notes on Music

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This Unit Problem 1 is based information you learned in Lessons 1, 2, and 3 of Unit 7. The problem involves gathering data related to the music industry for a school newspaper and visually representing this data in two graphs—one graph will represent the data accurately, and the other graph will misrepresent the data. As well, you will create suitable newspaper headlines for two articles based on the two graphs.

What data should you collect? Think of a statement that expresses your view relating to the music industry; for example, "Nelly Furtado is Canada's most popular musical artist." Then gather data that would support this view.

Then you represent the data by creating two graphs. One graph is to represent the data accurately and support your viewpoint. The other graph is to misrepresent the data so that it supports a view different from yours.

Finally, the headlines you suggest for the two newspaper articles should, in a nutshell, express the viewpoints that your graphs support.

Work with a partner. Your partner may be a classmate, friend, or someone from your discussion board.

Read

Go to "Wrap It Up!" on page 39 of the textbook to read a description of your Unit Problem 1. The final work you are to submit for Unit Problem 1 includes the following:

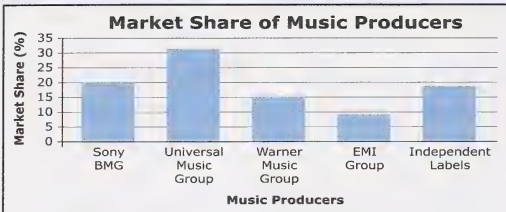
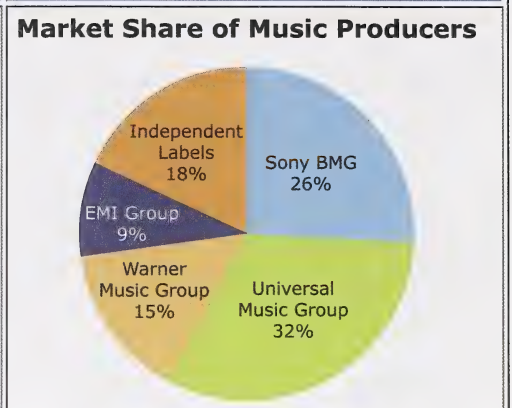
- a table showing the data you collected
- two graphs—one representing the data accurately, the other misrepresenting the data
- two headlines—one related to the graph that represents data accurately, the other related to the graph that misrepresents the data

For a sense of how data will be used and represented in Unit Problem 1, read "Math Link" on page 5 of the textbook.

Self-Check

SC 1. Complete "Math Link" questions 1.a), 2, and 3.b) on page 5. Discuss with a partner—possibly using instant messaging—how to use graphs and data tables to display information for music producers.

Note: Question 3.b) is based on the comparison of a bar graph and a circle graph for the data table. This data table gives the market shares of several music producers. The bar graph and circle graph are shown here.

Bar Graph	Circle Graph
	

Adapted from Bruce McAskill et al., *MathLinks 8: Teacher's Resource* (Toronto: McGraw-Hill Ryerson, 2008), 9. Reproduced by permission.

Compare your answers in the Appendix.

Try This

TT 1. Complete "Math Link" on page 17 of the textbook. This activity is about surveying students to find out the type of music preferred by other students. If you are not in a class, you may survey other Grade 8 students on your discussion board. Survey at at least 15 students. If you need to, ask your teacher for survey data.

My Guide

Q: What are some types of music?

A: Look at the circle graph on page 5 of the textbook. The types or categories of music shown on the graph are "Rock," "Country," "Classical," "Rap," and "Alternative." You may use a different list of music types that makes sense to you and to most of the students you will be surveying. The choice "none of the above" can be used to represent types of music that few students would likely pick.

Q: Which graph format can I use for my data?

A: See the different graphs you could use on page 12 of your textbook. The advantages of each type of graph are indicated.

Place a copy of your answers in your Math 8 course folder.

TT 2. Complete "Math Link" on page 27. This activity asks you to research music sales for several artists of your choice. Include one artist from the music type preferred by the students you surveyed.

Place a copy of your answers in your Math 8 course folder.

TT 3. Complete "Math Link" on page 35. You are asked compare and read two graphs representing related data.

Place a copy of your answers in your Math 8 course folder.

TT 4. Complete "Wrap It Up!" on page 39 of the textbook. You will research data related to a music-industry topic and visually display the information that you find. You may be able to use your research related to the popularity of different types of music or the music sales of various artists.

Place a copy of your answers in your Math 8 course folder. When you are satisfied with your answers to TT 4, submit them to your teacher.

Unit Problem 2: A Fair Chance

The Unit Problem 2 is based on information that was presented in Lessons 4, 5, and 6 of this unit. This unit problem has to do with a stick game based on traditional First Nations' recreational activities. In each turn of the stick game, a player drops a bundle of four flat sticks that are decorated on one side. In this game, points are earned according to the number of sides that face up.

Photo © Canadian museum of Civilization, III-G-556 a-d, photo D2004-10234

For this problem, you will play and analyze the stick game. You will study the sample space associated with the game. Then you will judge the fairness of the way points are earned. Then you will design a better scoring system that is fair to each player.

You will need a partner to carry out this unit problem. Your partner will be someone with whom you will discuss the application of probability concepts, who will help decorate the sticks for the stick game, and who will play the game of sticks with you.

Read

Go to "Wrap It Up!" on page 439 of the textbook to read a description of the core of Unit Problem 2. The problem is analyzing a traditional probability game, known as a stick game, and modifying the scoring system so each player earns points in a fair way. The final work you are to submit for the Unit Problem includes the following:

- a tree diagram showing outcomes for a drop of the sticks (The outcomes are to be shown as lists of Ds, for decorated side up, and Bs, for bare side up, and also as numbers indicating the number of decorated sides up.)
- the evaluation of the probability of each of the five possible numbers of decorated sides up
- an assessment of the fairness of the games scoring system
- a modified scoring system to improve the game

Read "Math Link" on page 409 of the textbook. In order to judge the fairness of a probability game, you will need to sense of what *likelihood* means and how cards and dice affect the outcome of games.

Self-Check

SC 2. Complete "Math Link" questions 1, 2, and 3 on page 409. With a partner, discuss how cards and dice are used in games. You may wish to use instant messaging when you are discussing SC 2 with your partner.

Compare your answers in the Appendix.

Read

Read the first paragraph of "Math Links" and "Did You Know?" on page 418 of your textbook. What was originally used instead of sticks in the stick game?

Try This

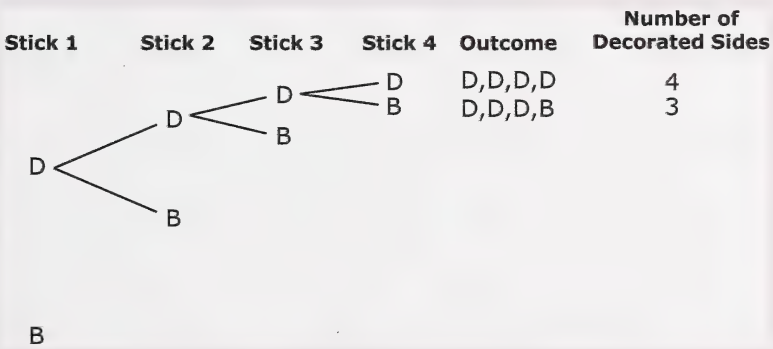
TT 5. Complete the "Math Link" questions on page 418. This activity asks you to use a tree diagram (or table) to determine the possible outcomes—that is, the sample space for the game.

My Guide

Q: Can a table be used to show the possible outcomes, or should a tree diagram be used instead?

A: Think of each landing of the four flat sticks as an independent event. A simple table can only handle two events. A tree diagram can be made for four simultaneous events. A tree diagram with four levels would work.

You can make the tree diagram start this way:



TT 6. Complete the "Math Link" questions on page 425, which asks you to create a different strategy for determining possible outcomes in the game.

TT 7. Complete the "Math Link" questions on page 435.

Place a copy of your answers in your Math 8 course folder.

Discuss and Share

For TT 8, follow the directions in "Wrap It Up!" on page 439; prepare the sticks and play the stick game with a partner. Make a set of sticks for a stick game by decorating something like Popsicle® sticks or tongue depressors. Set the number of stick drops to 50, and then play the game. After you have finished playing the game, discuss the fairness of the scoring. You may include an additional person from your discussion board to help you analyze the stick game.

TT 8. Complete "Wrap It Up!" questions a), b), and d) on page 439. When you respond to part a), you may use your answer to TT 5. In your answer to part b), include a modified scoring system that you think would more likely keep the scores closer throughout the game.

My Guide

Q: Does the tree diagram made for TT 5 show the sample space?

A: Yes, this diagram shows the outcomes and shows the outcomes as the number of decorated sides up. All four down is expressed by a zero in the column "Number of Decorated Sides."

Q: Can the tree diagram be used to figure out the theoretical probability of the outcomes?

A: Yes. Identify the favourable outcomes among the possible outcomes of the tree. Then find the theoretical probability using this formula:

$$\text{probability} = \frac{\text{number of favourable outcomes}}{\text{number of possible outcomes}}$$

The probability is theoretical because it was not found through experimentation, such as dropping the sticks to the ground repeatedly.

Q: What is the difference between a fair game and an unfair game of probability?

A: Suppose you played a simple game with another person. Each turn you selected a ball from a sack and put the ball back. You score 1 point if you pick a white ball and your opponent scores 1 point if he or she picks a black ball. The bag, however, contains one white ball and two black balls. Over several turns, your opponent would likely earn more points. This game is unfair because your opponent has a greater likelihood of picking a black ball than you do of choosing a white ball.

You can modify the game to make it fair. One way is to add a second white ball to the sack so that there is the same number of black balls and white balls.

The other modification you could make to the the game is to change the way you earn points by making it the same for both players: You could rule that whoever picks a white ball scores 1 point and whoever picks a black ball scores 0 points. Then each player has the same probability of winning and it's a fair game.

Q: In the game of sticks, a scoring system applies the same way to both players so the game is fair. Does that make the scoring system fair?

A: A fair game may not have a fair scoring system. A scoring system for a probability game is fair when it gives more points to outcomes that are unlikely and fewer for those that are likely. You can see this idea behind a scoring system used in games of skill, where the demonstration of great skill earns more points. In darts, for example, landing a dart in the small red circle at the centre of the board—the bull's eye—is worth more points than any other shot.

In a stick game, for example, you could improve the fairness of the scoring system by awarding six times as many points for an outcome that is just one-sixth as likely to occur as an outcome that you would award 1 point for.

Place a copy of your answers in your Math 8 course folder. When you are satisfied with your answers, submit them to your teacher.

Unit Summary

Congratulations on finishing all of the lessons in Unit 7! In this unit you investigated the use of graphs to represent data and developed strategies to determine the probability of events.

You found that you could represent data using bar graphs, double bar graphs, circle graphs, line graphs, and pictographs. You also found that just because data is represented in a graph does not mean that data is presented neutrally. There are ways to format a graph so that it misrepresents data and is misleading. Knowing this enabled you to critique graphs for how well they represent data.

Probability can be determined by counting the favourable events in a sample space and comparing the number counted to the total number of events in the sample space.

In a probability experiment involving two or more independent events, the total number of outcomes in a sample space can be determined by multiplying the number of outcomes for each of the independent events. The probability of success for the experiment can be determined by multiplying the probabilities of the individual events.

You also applied the knowledge and skills of this unit in reporting on the music scene and analyzing a probability game.

Unit Review

This unit was based on "Chapter 1: Representing Data" on pages 2 to 41 of the textbook and "Chapter 11: Probability" on pages 406 to 441. Your unit review will be based on the Chapter 1 and Chapter 11 reviews.

Self-Check

SC 3. Turn to "Chapter 1 Review" on pages 36 and 37 of the textbook. Complete questions 1, 2, 3, 4, and 5. Then answer at least one question from each of the three lesson components. You may do additional questions for practice.

SC 4. Turn to "Chapter 11 Review" on pages 436 and 437 of your textbook. Complete questions 1, 2, 3, 4, and 5. Then answer at least two questions from each of the three lesson components. You may do additional questions for practice.

Compare your answers in the Appendix.

Are You Ready?

You've done a lot of work to reach this point in the unit. But are you ready to strut your stuff—that is, show your mastery of the new concepts and skills? Are you really ready to take the challenge of a unit test?

You may feel you are ready but you want to do a practice test before doing one for marks. Well, here's your chance.

This unit was based on two chapters in your textbook. The following Self-Check questions consist of selected questions from the two textbook practice tests.

Challenge yourself and complete the Self-Check questions as you would a real unit test. At the end, see how well you do when you compare your answers. Review any unit sections you still might have questions about at this point.

Self-Check

SC 5. Complete the odd-numbered questions of "Chapter 1 Practice Test" on pages 38 and 39 of the textbook.

SC 6. Complete the odd-numbered questions of "Chapter 11 Practice Test" on pages 438 and 439 of the textbook.

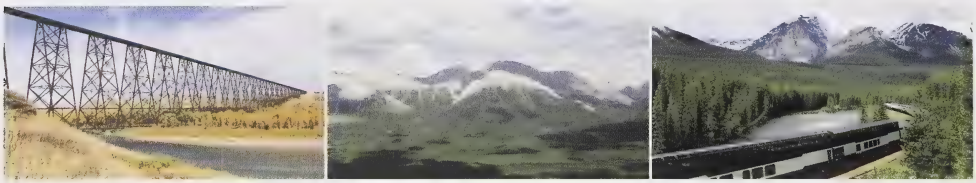
Compare your answers in the Appendix.

Assignment

Check with your teacher about a unit test.

Congratulations! You've done it again: You have finished another unit of study. Way to Go! You've earned it: Go to the Math 8 Multimedia DVD, and open the item titled "Applause."

Mathematics 8 Summary



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Think back over your learning journey as you progressed through this math course. You should now be able to credit yourself with a range of achievements:

- Improving your number sense through a study of squares, square roots, percent, rate, ratio and operations with fractions and integers.
- Using linear relations and their graphs to describe real-world situations and to solve problems.
- Using linear equations of various forms to model and solve problems.
- Using the Pythagorean theorem and determining the surface area and volume of 3-D objects to solve problems.
- Using top, front and side views to describe the characteristics of 3-D objects.
- Critiquing circle graphs, line graphs, bar graphs and pictographs for the way they displayed data.
- Using experimental and theoretical probabilities to represent and solve problems involving independent event

So, congratulations, you have completed Mathematics 8 EveryWare!

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Contact your teacher to find out if there's a final test or any additional assignment for you to complete and submit for marking.

Appendix

Lesson 1

SC 1.

1. The bar graph is easier to read because no calculations are required.

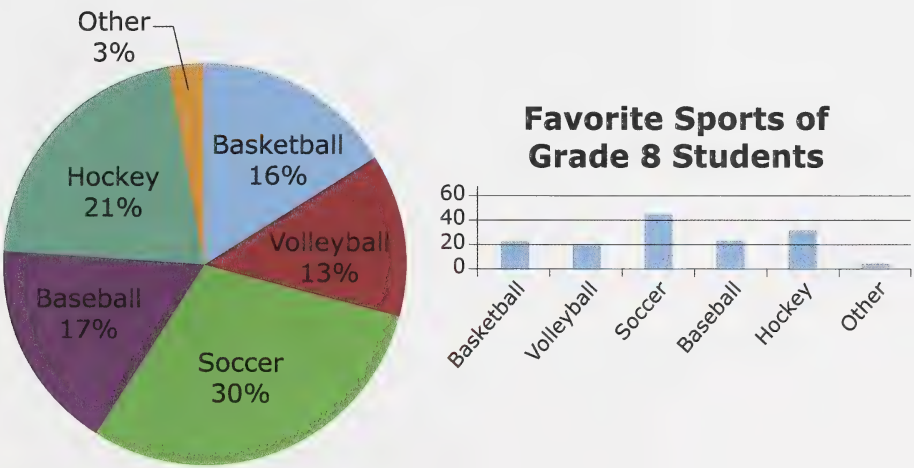
2. A line graph would be useful because the apple varieties are completely independent. Line graphs are better suited to showing changes in data over time.

3. A circle graph could be used.

Bruce McAskill et al., *MathLinks 8: Teacher's Resource* (Toronto: McGraw-Hill Ryerson, 2008), 14. Reproduced by permission.

SC 2.

a. A circle graph or a bar graph would be suitable. A line graph is not because it suggests that changes occur over time. A pictograph would involve drawing parts of a picture symbol for small amounts. Parts of a picture symbol would be hard to draw accurately and be difficult to interpret.



b. Answers will depend on the type of graph you chose to use in order to display the data. The circle graph makes it easy to see how a category relates to the whole. The bar graph makes it easy to compare data across the sport categories.

SC 3.

4.a. Ravi spends $50 on food and $40 on movies. Therefore, he spends $50 – $40 = $10 more on food. The pictograph shows this more clearly. In the pictograph, each symbol represents $10. Five symbols were used for food, and four symbols were used for movies. No partial symbols were used. In the circle graph, the percents must be multiplied by $200 to determine the amounts spent on food and movies.

b. The circle graph shows that half of the circle is made up of the categories "Food" and "Clothing." Clothing is 25% and food is 25%, 25 % + 25% = 50%.

c. Answers may vary. Example: One advantage of the circle graph is that the categories are represented as percents of the total spending. One advantage of the pictograph is it is a precise way of presenting each amount, since the amounts were all multiples of ten. One disadvantage of the circle graph is that it is necessary to perform a computation to determine the actual amounts spent. One disadvantage of the pictograph is that the percent of money spent on each category must be calculated.

6.a. The line graph shows the change of growth from week 1 to week 4. The pictograph shows the height of the plant at the end of weeks 1, 2, 3, and 4.

b. Between weeks 1 and 2, the plant grew 20 cm (40 – 20 = 20). Between weeks 2 and 3, the plant also grew 20 cm (60 – 40 = 20).

c. The plant changed in growth 20 cm (40 – 20 = 20) between weeks 1 and 2. Between weeks 2 and 3, it changed in growth 20 cm (60 – 40 = 20). Between weeks 3 and 4, the plant changed in growth 30 cm (90 – 60 = 30). Between weeks 3 and 4, it changed the most in height. The line graph shows this more clearly.

d. One advantage of the line graph is that it shows the rate of change of the growth of the plant. One advantage of the pictograph is that it shows the height of the plant after each week. One disadvantage of the pictograph is that it does not show the rate of change. There is no disadvantage to the line graph.

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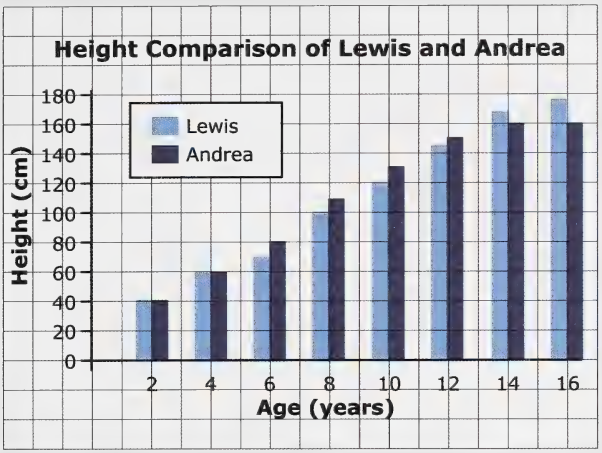
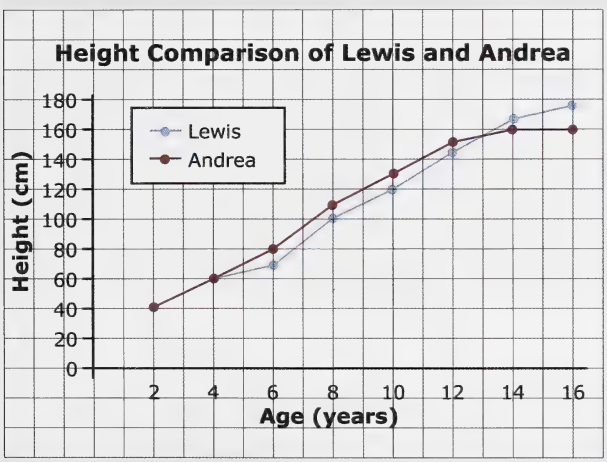
SC 4.

7.a.



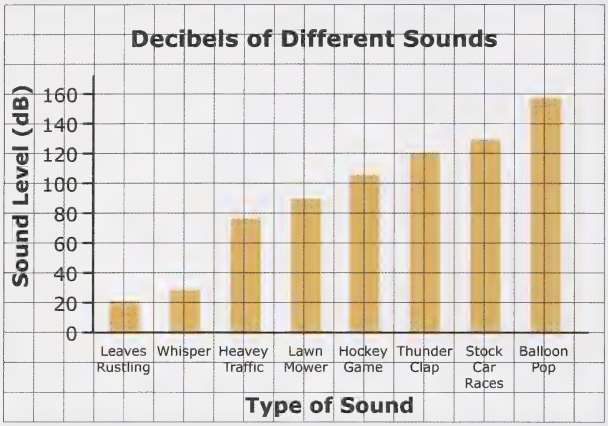


b. The trends are similar in that both friends' heights are increasing from age 2 to age 12. The trends are different in that Andrea's height stops increasing after age 14, but Lewis's height continues to increase after age 14.

c. The line graph shows their height trends more clearly because it shows changes in their heights over time.

d. The heights are not parts of a whole. Therefore, the data would not be suitable for a circle graph.

8.a.



A bar graph allows for an accurate comparison of sound levels.

b. The data are not a comparison over a period of time; therefore, a line graph would not be suitable.

c. The data do not represent a comparison of parts to a whole; therefore, a circle graph would not be suitable.

d. A pictograph could be used, but it would not be as accurate as a bar graph because of the fractions of symbols required to represent these data.

MathLinks 8: Solutions CD (Toronto: McGraw-Hill Ryerson, 2008) 4-5. Reproduced by permission.

SC 5.

12.a. Answers may vary. Sample answer: The data range from 40 to 98, which allows for six intervals of ten points each. Having the intervals start or end on multiples of ten can make their placements on the horizontal axis of a grid easier. Having the intervals too narrow may keep the frequencies so low so that the reader does not see where most of the students score. Having the intervals too wide can put so many in one category that it's hard to pinpoint where the most common scores fall.

Make sure your frequencies add up to 30 and your intervals.

The number of scores in each interval were counted and recorded in the table.

Math Test Scores	
Range	**Frequency**
40 to 49	4
50 to 59	4
60 to 69	5
70 to 79	8
80 to 89	5
90 to 99	4
Total	30

b. A bar or circle graph could be used.

c. A bar graph would give the number of scores in each range. A circle graph would compare the categories to the whole using percent.

14.a. A bar graph or circle graph would be more appropriate for the data.

b. A line graph shows change over time. This survey does not apply to that type of data.

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Going Beyond

17.a. Answers may vary. A question that has just a few possible responses works well. Example: A survey question to ask members of your class could be, "What is your favourite season of the year: winter, spring, summer, or fall?" There are four choices. There is a good chance that there will be different numbers of people selecting each option.

b. Since the total number of people responding to the question will be known, the percent for each response can be calculated.

c. Answers may vary. A question that has many possible answers would be best for a graph other than a circle graph. A question that has a numerical value as an answer has many possible answers. Example: A survey question to ask members of your class could be, "How many hours a week do you spend watching television?" or "How much money do you have in your bank account?"

A bar graph or pictograph may be more suitable for this data.

MathLinks 8: Solutions CD (Toronto: McGraw-Hill Ryerson, 2008) 11. Reproduced by permission.

Lesson 2

SC 1.

1. Your list of similarities should include the following:

- The graphs display the same data.
- The graphs have the same title.
- The vertical axes on both graphs are labelled "Quantity," and the horizontal axes are labelled "Month" on both graphs.

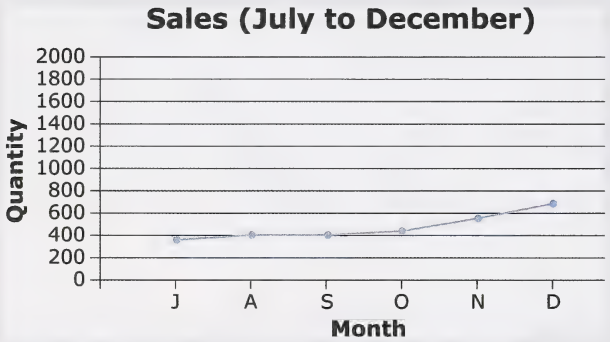
2. Your list of differences should mention that the graphs use different scales on the vertical axis. More specifically, the scales differ in these ways.

- Graph A uses a continuous scale, while Graph B has a break in the scale.
- Graph A uses a scale that increases by 100 units at a time, while Graph B starts at 350 and increases by 50 units at a time.
- Graph A uses a scale that spans 800 units, while Graph B uses a scale that spans just 350 units.

3. Graph A gives a more accurate representation since it uses a continuous scale that starts at zero.

Keep in mind that using a break symbol in the scale is usually not done to misrepresent data, but is usually done for practical purposes instead. For example, you might use a break to fit all the data points on one graph. The break may allow you to use a scale that would make differences in values easier to see on the graph. In science, you may have had to use a break when working with very large (or very small) numbers to show experimental results clearly.

4.



Bruce McAskill et al., *MathLinks 8: Teacher's Resource* (Toronto: McGraw-Hill Ryerson, 2008), 21. Reproduced by permission.

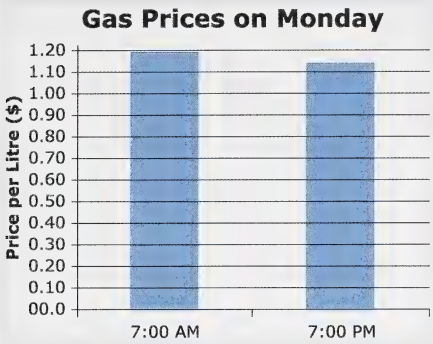
SC 2.

a. The vertical axis was drawn with a break in the scale that makes the decrease in gas price seem more significant than it is.

b. The graph makes it appear that the price of gas is half as much as it was at the beginning of the day.
c. The graph could be redrawn with a continuous scale that starts at $0.00 and goes to just above $1.19 so that the price difference would still be easily seen.

For example, the scale could span $0.00 to $1.20, with intervals of ten cents.

Such a graph could look like this:



Bruce McAskill et al., *MathLinks 8: Teacher's Resource* (Toronto: McGraw-Hill Ryerson, 2008), 22. Reproduced by permission.

SC 3.

a. The symbols in the pictograph are not the same size, and they are not spaced evenly.

b. It appears that an equal number of people prefer each of the three lunch specials.

c. Draw the graph with symbols of equal size, spaced an equal distance apart.

Bruce McAskill et al., *MathLinks 8: Teacher's Resource* (Toronto: McGraw-Hill Ryerson, 2008), 22. Reproduced by permission.

SC 4.

a. The area of the large circle appears to be four times the area of the small circle. The greater area of the large circle suggests that the costs have greatly increased.

b. The cost in 2007 appears to be four times the cost in 1997, even though it is actually only two times the cost.

c. Answers will vary. Example: Use bars of the same width instead of circles.

Bruce McAskill et al., *MathLinks 8: Teacher's Resource* (Toronto: McGraw-Hill Ryerson, 2008), 22. Reproduced by permission.

SC 5.

5.a. The column representing B appears to be about three times as tall as the column representing A.

b. There were 200 votes for B and 100 votes for A; 200 ÷ 100 = 2. Therefore, B received two times as many votes.

c. Answers may vary. Example: It appears that candidate B received three times as many votes as candidate A.

d. The graph could be redrawn with the vertical axis showing values from 0 to 200, with no break.

6.a. Apples seem to sell the best because the line containing the apple symbols is longer than the lines for the other fruits.

b. Answers may vary. Example: It appears that about the same number of each fruit was sold because the line representing each type of fruit is approximately the same length.

c. The pictograph should be redrawn so that each symbol is the same size, and the symbols should be spaced the same distance apart.

8.a. Answers may vary. Example: The horizontal scale has a break from levels 0 to 10. Also, the width of each bar is not equal.

b. Scott's progress appears to be more than twice as much as Bryce's progress.

c. Start the scale at zero with no break between levels 0 and 10, and make Scott's and Bryce's bars have equal widths.

9. Answers may vary. Example:

a. The vertical axis in Graph A has uniform scaling from 0 to 160 by units of 20. The vertical axis in Graph B has a break between 0 and 140, after which the scale increases uniformly by units of 2.

b. In Graph A, it appears that the sales were very near the same amount for each student. In Graph B, it appears that Chris sold twice as many bars as Megan.

c. Graph A is more accurate because the vertical scale has no break.

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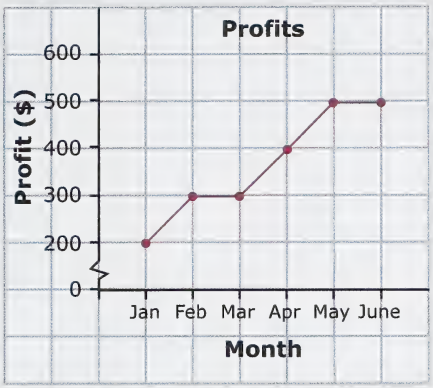
SC 6.

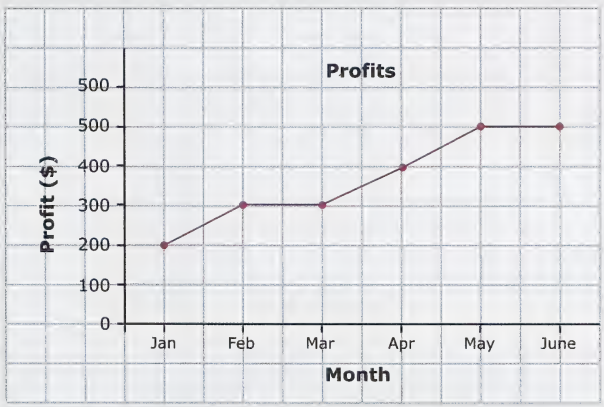
10. Answers may vary.

a. The vertical scale has a break between 100 and 200. This break makes the profit increase look larger than it would look on a graph with no break.



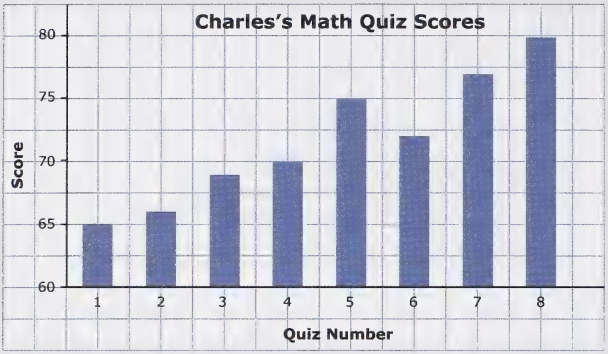
b. The profits seem to be four times as high in June as they were in January.

c. The profits for June were $500 and the profits for January were $200; $500 divided by $200 equals 2.5. Therefore, the profits for June were two and a half times the profits for January.

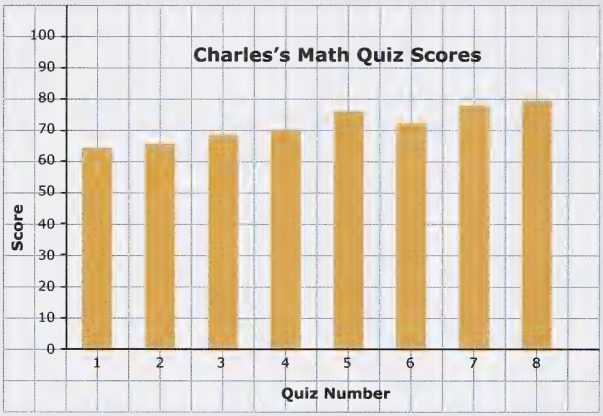


11. Answers will vary.

a. Put in a break on the vertical axis so that you in effect start the scale of the vertical scale at 60%. The following is a sample bar graph.



b.



c. In Graph A, it appears that Charles's scores have been increasing by a large amount. In Graph B, the increase in scores does not seem as large.

12.a. The vertical scale goes from 0 to 500, but the greatest number of votes was less than 200. Therefore, it appears that all three candidates were close to winning, and the title implies that all three candidates tied.

b. Answers may vary. Example: Candidate B barely won the election.

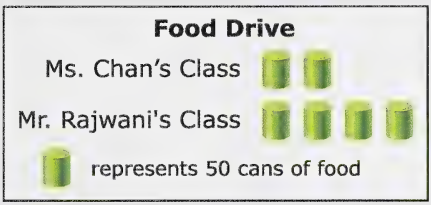
c. No, the votes were not divided evenly three ways. Answers may vary. Example: A new title could be "Election Results."

13.a. The information does not appear to be the same because the sector sizes in the graphs are different.

b. The sizes for the sectors in Graph A do not appear to coincide with their percents. For example, in Graph A, sector D is half of the circle, but it is labelled with only 35%. There are two pieces in Graph A that are each 25% (the green and the yellow), but they appear as different size wedges.

14.a. There is no scale or legend for comparison purposes. The volume of the box for Mr. Rajwani's class is much greater than two times that of the box for Ms. Chan's class. Also, the number of cans in each box is equal.

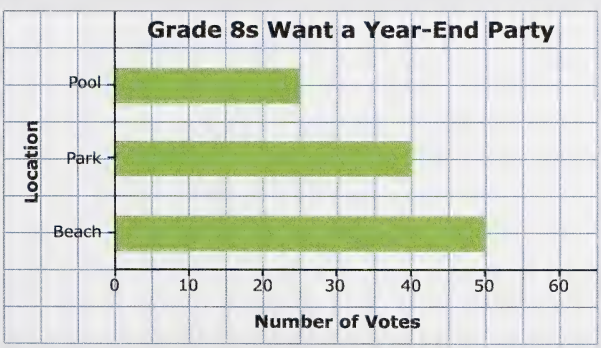
b. The graph is hard to interpret but the larger box has two times the width, height, and length of the smaller box. So the larger box has eight times the volume. Mr. Rajwani's class appears to have collected eight times the amount of food that was collected by Ms. Chan's class.



15.a. The beach appears to be about three and a half times as popular as the pool.

b. No, the majority did not choose the beach. There were 25 + 40 + 50 = 115 votes cast; 50 out of 115 is approximately 43%. A majority is 50% or more.

c. Draw a graph that does not have a break in its axis. For example, the following bar graph represents the data properly.



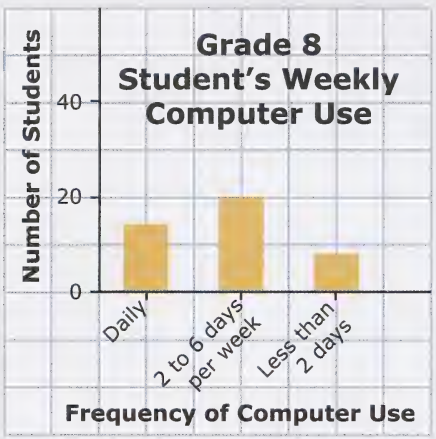
16.a. Cool Flavours probably created this graph because it appears that they sell twice as much ice cream as Dairy Tasty.

b. The main problems are these:

- There is no scale for comparison purposes.
- The cones vary in width as well as height exaggerating the visual impact of the visuals.

Comment: You may find these comparisons in percentage terms informative. The difference between the two shops is 0.2 million. The difference is only 4% based on the Cool Flavours sales (0.2/4.5 ×100% = 4%). But the Dairy Tasty image appears about 50% smaller.

18. Answers may vary. Example:



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Going Beyond

Here are some differences you may have included in your answer.

- On the Mollweide world map, the size of Greenland and the northern land areas are smaller than the same regions shown on the Mercator world map.
- The Mollweide world map has an elliptical shape, whereas the Mercator world map is rectangular.
- The Mollweide world map shows Antarctica, but the Mercator world map does not.
- On the Mollweide world map, Europe appears much smaller in comparison to Africa than it does on the Mercator world map.
- On the Mercator world map, North America and South America are both more "upright" than on the Mollweide world map, where there seems to be a bend in Central America.

Both maps are similar in that they both show all the land continents other than Antarctica.

Lesson 3

SC 1.

1. The author may have used a kind of stacked graph because there were two sets of data across the categories for years.

You may have considered this an effective graph, but this graph can be confusing to those not familiar with this kind of graph. The first impression is that only the bar visible above the shorter bar represents the quota. But the quota is represented by the whole bar, which includes the visible part as well as the part hidden by the bar for "Catch."

2. The graph is not misleading.

a. The graph is not misleading because there are no breaks in the scale, the scale is uniform, and the visual elements are properly sized. More specifically, the graph uses a continuous scale starting at 0, with intervals of 50 units on the vertical axis. There is a uniform scale of 1-year units from 1995 to 2005, with no break, on the horizontal axis.

b. You should use a double bar graph to show quota and catch beside each other in each year. This would make the graph easier to understand.

3.a. The catches were significantly lower from 1995 to 2001, when the coded-wire tags were used.

b. The graph shows that the catches were significantly higher from 2002 to 2005, when DNA testing was used.

Bruce McAskill et al., *MathLinks 8: Teacher's Resource* (Toronto: McGraw-Hill Ryerson, 2008), 29. Reproduced by permission.

SC 2.

a. Answers will vary. Example: The circle graph compares the percent of teens that picked a specific number of baskets to the total number of teens.

b. The graph is not misleading. Explanations may vary. Example: It indicates the total number of teens, represents each percent accurately, and the percents total 100%.

c. Look for two accurate conclusions. Examples:

- The majority of teens picked between 4 and 6 baskets.
- The same number of teens picked 4 baskets as 6 baskets.

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SC 3.

4.a. There are 2 sets of data across several categories. Madison used a double bar graph to compare each type of communication by gender.

b. The graph is not misleading. It is scaled uniformly with no breaks, and the bars are a consistent width.

c. Answers may vary. Example: More girls use the Internet than boys. Both boys and girls prefer to communicate in person or on the Internet.

d. The data do not show a change over time.

5.a. Answers may vary. Example: More Grade 9 students spend less than 1 h on the Internet than Grade 8 students. Twenty-four Grade 8 students spend 2 h to 3 h on the Internet.

b. No, the graph is not misleading. The scale is uniform, there are no breaks, and both axes are clearly labelled.

c. No major improvements are needed, but colours that stand out beside each other would have improved the graph, instead of the two shades of purple (e.g., pretty similar colours).

d. A comparison of time spent on the Internet of both grades can be shown on just one graph.

e. Two circle graphs (one for Grade 8 students, one for Grade 9 students) would show the percent of the students in each grade who use the Internet for each time interval. The circle graphs may be more informative.

Comment: Sometimes there is more than one graph that can work equally as well. Then it comes down to personal preference and how easy or difficult the graphs are to create.

6.a. Answers may vary. Example: Eighteen cars in the parking lot were sedans; 48% of the cars in the parking lot were either SUVs or minivans (SUVs is 28% and minivans is 20%; 28% + 20% = 48%).

b. The graph is not misleading. The sections of the circle are labelled with the category name and percent. The title contains the total number of cars in the lot.

c. Answers may vary. Example: A circle graph shows a comparison of each car type as a percent of the total number of cars.

d. Answers may vary. Example: A bar graph could have been used to show a comparison of the number of cars in the lot. However, it would not have shown what percent of the cars in the lot was each type of car.

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SC 4.

7. Answers may vary.

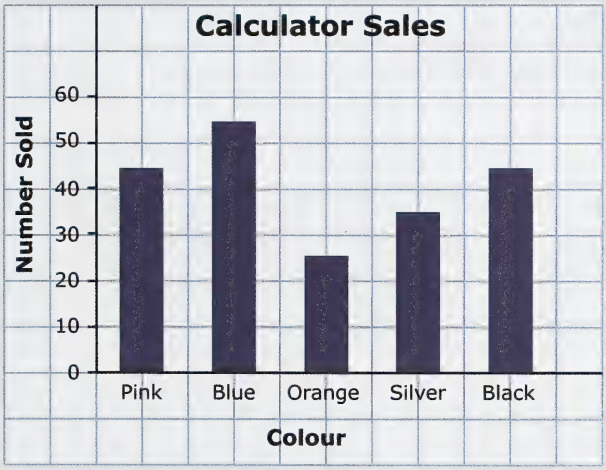
You could use the bar graph. The bar graph shows that "Health" has almost twice as many votes as any other type of organization. So "Health" deserves your donation.

You could also use the circle graph. It shows that over $\frac{1}{3}$ of students would like to support "Health." This also shows that many students feel that "Health" deserves your donation.

Comment: Perhaps you are from the Environment Foundation. The bar graph shows that your donations are much less than any other organization. You might be able to use this graph to gain the sympathy donations.

8.a. The size of the bar for the blue calculators appears to be about three times as large as the bar for the orange calculators, but Truong is not correct. The scale indicates that 56 blue calculators were sold and 25 orange calculators were sold: $56 \div 25$ is approximately 2. His graph is misleading because there is a break in the *y*-axis.

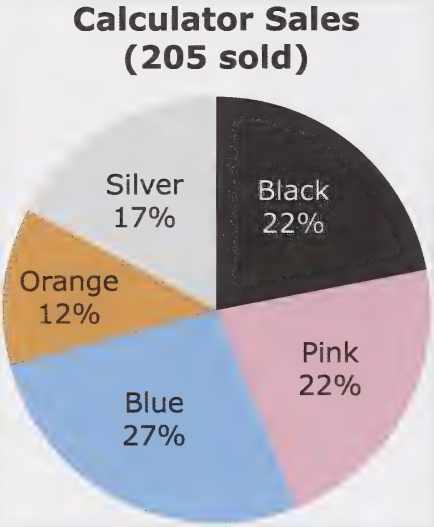
b.



Conclusions may vary. Example: About twice as many blue calculators as orange calculators were sold.

Comment: Colouring the bars in the graph the colour of the calculator that they represent would make the graph easier to read.

c.



d. A circle graph shows the percent of calculators sold for each colour. This indicates directly how each category compares to the total number sold.

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Going Beyond

11.a. Graph A makes it appear that there was a small change in minimum wage from 2000 to 2007. Graph B makes it appear that there was a large change in minimum wage from 2000 to 2007.

b. Graph A would support such a claim. The minimum wage has increased from a little over $6 to $8 in seven years.

c. An employer would use Graph B because it appears that there has been a large increase in the minimum wage over the seven-year period.

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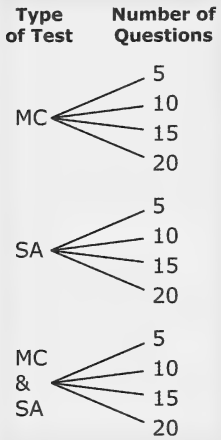
Lesson 4

SC 1.

1. Your answer should look like one of the following:



		Number of Questions			
		5	**10**	**15**	**20**
Type of Test	MC	MC, 5	MC, 10	MC, 15	MC, 20
	SA	SA, 5	SA, 10	SA, 15	SA, 20
	MC & SA	MC & SA, 5	MC & SA, 10	MC & SA, 15	MC & SA, 20

2. There are three equally likely outcomes for the type of test. Just one of the three outcomes is multiple-choice test, so the probability is one out of three or $\frac{1}{3}$. The probability that the test will be multiple choice is $\frac{1}{3}$.

3. The probability of ten questions occurring on the test is $\frac{1}{4}$. There are four equally likely outcomes for the number of questions, but only one of these outcomes is ten questions.

Note: $P(10 \text{ questions}) = \frac{1}{4}$ is a mathematical notation for the sentence, "The probability of ten questions occurring on the test is $\frac{1}{4}$."

4. (MC, 5), (MC, 10), (MC, 15), (MC, 20), (SA, 5), (SA, 10), (SA, 15), (SA, 20), (MC & SA, 5), (MC & SA, 10), (MC & SA, 15), (MC & SA, 20)

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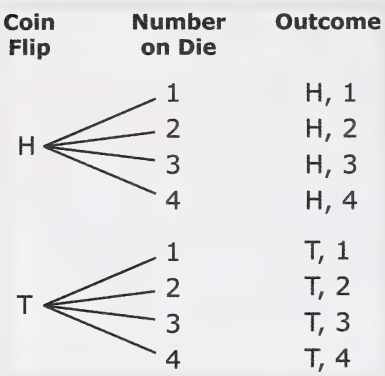
SC 2.



Bruce McAskill et al., *MathLinks 8: Teacher's Resource* (Toronto: McGraw-Hill Ryerson, 2008), 556. Reproduced by permission.

SC 3.

a.

		Die					
		1	2	3	4	5	6
Spinner	1	1, 1	1, 2	1, 3	1, 4	1, 5	1, 6
	2	2, 1	2, 2	2, 3	2, 4	2, 5	2, 6
	3	3, 1	3, 2	3, 3	3, 4	3, 5	3, 6
	4	4, 1	4, 2	4, 3	4, 4	4, 5	4, 6

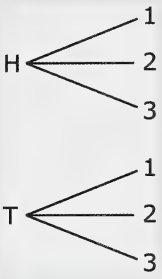
b. $P(4,4)=\frac{1}{24}$

c. $P(\text{sum}>5)=\frac{7}{12}$

Bruce McAskill et al., *MathLinks 8: Teacher's Resource* (Toronto: McGraw-Hill Ryerson, 2008), 556. Reproduced by permission.

SC 4.

3.a. There are two outcomes (heads or tails) for the coin and three outcomes for the spinner (1, 2, and 3).



b. The sample space is listed as ordered pairs in brackets from the outcomes listed in a): (H, 1), (H, 2), (H, 3), (T, 1), (T, 2), and (T, 3).

c. Each of the six possible outcomes of both events (the coin flip and the spin) is equally likely.

$$P(\text{H}, 2) = \frac{1}{6}$$

The probability of $P(\text{H}, 2)$ is $\frac{1}{6}$ or $0.1\overline{6}$ or $16.\overline{6}\%$.

5.a. Organize the four outcomes for the green die vertically on the left side of the table. Place the four outcomes for the blue die along the top of the table. List the sample space in the table.

		Blue Die			
		1	2	3	4
Green Die	1	1, 1	1, 2	1, 3	1, 4
	2	2, 1	2, 2	2, 3	2, 4
	3	3, 1	3, 2	3, 3	3, 4
	4	4, 1	4, 2	4, 3	4, 4

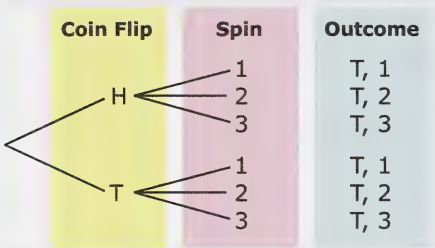
b. To determine P (sum > 5), count the favourable outcomes from the table and divide this by the possible outcomes. Since you want the outcomes with a sum that is greater than 5, you have to add the two rolls together to find which combinations give a total that is greater than 5.

The favourable outcomes are (2, 4), (3, 3), (3, 4), (4, 2), (4, 3), and (4, 4). There are six favourable outcomes and a total of 16 possible outcomes.

$$P(\text{sum} > 5) = \frac{6}{16}$$
$$= \frac{3}{8} \text{ or } 0.375 \text{ or } 37.5\%$$

c. There are four outcomes in the sample space that have the same number on both dice: (1, 1), (2, 2), (3, 3), and (4, 4)

$$P(\text{same number on both}) = \frac{4}{16}$$
$$= \frac{1}{4} \text{ or } 0.25 \text{ or } 25\%$$



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SC 5.

7.a. Arrange the four equally likely outcomes for the first catch along the left edge of the table, and then repeat the process along the top of the table for the second catch. Fill in the cells.

		Second Catch			
		W	T	C	Lost
First Catch	W	W, W	W, T	W, C	W, Lost
	T	T, W	T, T	T, C	T, Lost
	C	C, W	C, T	C, C	C, Lost
	Lost	Lost, W	Lost, T	Lost, C	Lost, Lost

b. There are 16 possible outcomes. There are two favourable outcomes that result in a whitefish (W) and a char (C) being caught.

$$P(\text{W and C}) = \frac{2}{16}$$
$$= \frac{1}{8} \text{ or } 0.125 \text{ or } 12.5\%$$

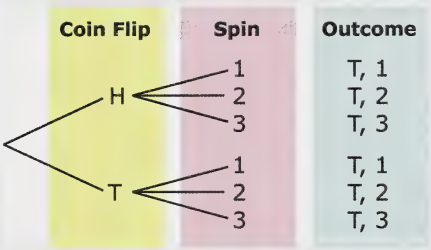
c. There is one favourable outcome that results in two char being caught.

$$P(\text{char, char}) = \frac{1}{16} \text{ or } 0.0625 \text{ or } 6.25\%$$

d. There is one outcome in the table that results in nothing being caught.

$$P(\text{lost, lost}) = \frac{1}{16} \text{ or } 0.0625 \text{ or } 6.25\%$$

8.a. There are two choices for the coin flip and five choices for the card selection.



b. Place the five cards along the top of the chart and the two coin flip outcomes along the left edge of the table.

		Card Chosen				
		6	7	8	9	10
Coin Flipped	H	H, 6	H, 7	H, 8	H, 9	H, 10
	T	T, 6	T, 7	T, 8	T, 9	T, 10

c. There are six favourable outcomes out of ten possible outcomes.

$$P(\text{outcome includes an even-numbered card}) = \frac{1}{6}$$
$$= \frac{3}{5} \text{ or } 0.6 \text{ or } 60\%$$

9.a. Place the two outcomes for the first baby along the left edge of the table and the two outcomes for the second baby across the top of the table.

		Second Baby	
		B	G
First Baby	B	B, B	B, G
	G	G, B	G, G

b. There are two favourable outcomes that result in one boy and one girl: a boy followed by a girl and a girl followed by a boy.

$$P(\text{one boy and one girl}) = \frac{2}{4}$$
$$= \frac{1}{2} \text{ or } 0.5 \text{ or } 50\%$$

c. It assumed that it is equally likely that a boy or girl is born for any birth, and that the two births are independent events.

10.a. Record two rows and columns for an E in order to say that each outcome in the table is equally likely.

		Second Spin			
		T	E	E	N
First Spin	T	T, T	T, E	T, E	T, N
	E	E, T	E, E	E, E	E, N
	E	E, T	E, E	E, E	E, N
	N	N, T	N, E	N, E	N, N

b. There are two outcomes that are favourable for spinning a T and an E, out of 16 possible outcomes.

$$P(\text{T then E}) = \frac{2}{16}$$
$$= \frac{1}{8} \text{ or } 0.125 \text{ or } 12.5\%$$

c. There are four favourable outcomes in the table.

$$P(\text{E, E}) = \frac{4}{16}$$
$$= \frac{1}{4} \text{ or } 0.25 \text{ or } 25\%$$

d. There are six favourable outcomes that result in the same letter appearing on both spins.

$$P(\text{same letter on both spins}) = \frac{6}{16}$$
$$= \frac{3}{8} \text{ or } 0.375 \text{ or } 37.5\%$$

MathLinks 8: Solutions CD (Toronto: McGraw-Hill Ryerson, 2008) 557-560. Reproduced by permission.

Going Beyond

2. Count the number of favourable outcomes by examining each of the eight possible pathways. There are three favourable outcomes.

$$\text{probability} = \frac{\text{number of favourable outcomes}}{\text{number of possible outcomes}}$$
$$= \frac{3}{8}$$

The probability of flipping exactly two heads and one tail (in any order) is $\frac{3}{8}$.

11.a. There are two possible outcomes, which are equally likely, at the top of the mountain. Therefore, the probability that they will take Thunder Road is

$$P(\text{Thunder Road}) = \frac{1}{2} \text{ or } 0.5 \text{ or } 50\%$$

b. Each of the eight lower runs is equally likely to be chosen. Two out of eight runs contain the name *Bowl*.

$$P(\text{skiing on a run that contains the name } \textit{Bowl}) = \frac{2}{8}$$
$$= \frac{1}{4} \text{ or } 0.25 \text{ or } 25\%$$

c. If the skier finishes on Quick Break, then he or she must have taken Thunder Road. Therefore, the probability of the skier taking both runs is the same as the probability of taking Quick Break.

$$P(\text{Thunder Road and Quick Break}) = \frac{1}{8} \text{ or } 0.125 \text{ or } 12.5\%$$

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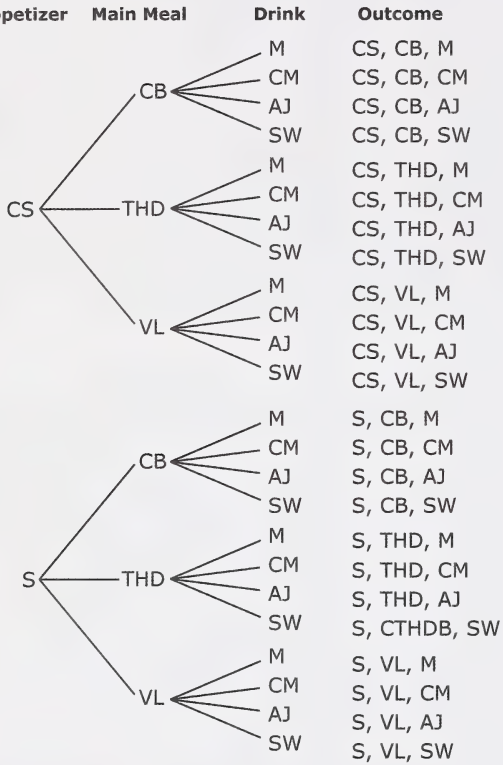
Lesson 5

SC 1.

1.



You could also use two tables—one for the choice of chicken soup and one for salad. Then main meals and drinks can be fitted along the left (column) and top (row) of the tables. Considered together, the tables should show the same outcomes as the tree diagram.

Chicken Soup Table

		Drinks			
		Milk	Chocolate Milk	Apple Juice	Water
Main Meal	Cheeseburger				
	Turkey Hot Dog				
	Vegetable Lasagna				

Salad Table

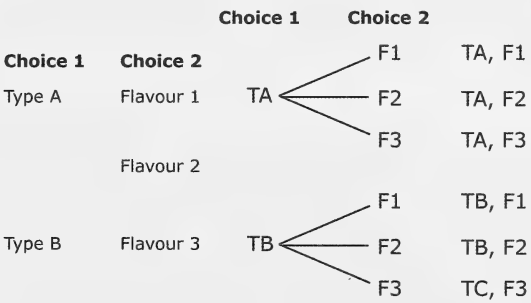
		Drinks			
		Milk	Chocolate Milk	Apple Juice	Water
Main Meal	Cheeseburger				
	Turkey Hot Dog				
	Vegetable Lasagna				

3. The total number of outcomes Sarah had was 24.

SC 2.

4. You may use your own strategies.

Part 1



There are six cone–flavour combinations.

Part 2

	Pants 1	Pants 2	Pants 3	Pants 4
Shirt 1	S1 P1	S1 P2	S1 P3	S1 P4
Shirt 2	S2 P1	S2 P2	S2 P3	S2 P4
Shirt 3	S3 P1	S3 P2	S3 P3	S3 P4
Shirt 4	S4 P1	S4 P2	S4 P3	S4 P5
Shirt 5	S5 P1	S5 P2	S5 P3	S5 P6

There are 20 shirt–pant combinations.

Part 3

5 models × 3 colours → 15

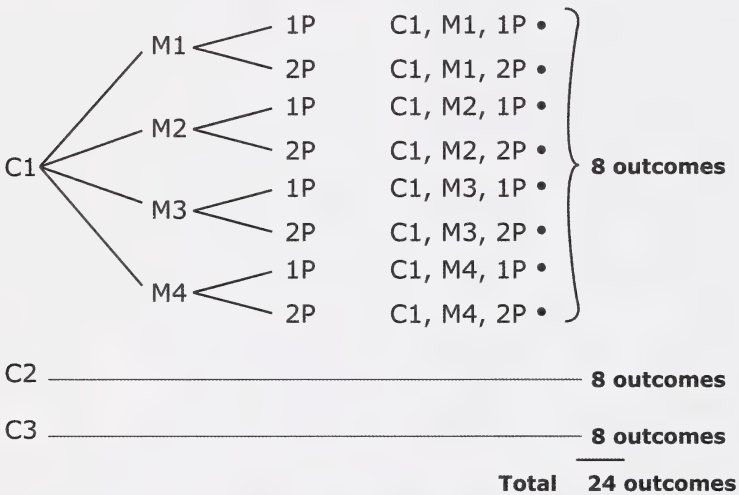
There are 15 sports car–colour combinations.

Part 4

4 computer models × 4 monitor modes → 16

There are 16 computer–monitor combinations.

Part 5



There are 24 computer–monitor–printer combinations.

5. For each row of the table, you can calculate the number of choices by multiplying the number of choices for each item. But you have to ignore none or interpret none as 1 for this to work in this table.

SC 3.

a. The table shows 12 possible sandwich combinations.

		Sandwich			
		Egg (E)	Tuna (T)	Ham (T)	Turkey (TY)
Bread	White (W)	W, E	W, T	W, H	W, TY
	Rye (R)	R, E	R, T	R, H	R, TY
	Whole Wheat (WW)	WW, E	WW, T	WW, H	WW, TY

b. Multiply the number of possible outcomes of sandwiches, four, by the number of possible outcomes of bread, three. The total number of possible outcomes is $4 \times 3 = 12$.

Alternately, you could use a tree diagram to get the same number.

Bruce McAskill et al., *MathLinks 8: Teacher's Resource* (Toronto: McGraw-Hill Ryerson, 2008), 566. Reproduced by permission.

SC 4.

a. and b. There are 18 possible combinations. You could determine this answer as follows:

- Draw a tree diagram to show 18 possible outcomes.

- Using multiplication, multiply the number of possible outcomes of sandwiches, three, by the number of possible outcomes of bread, three, by the number of possible outcomes of side orders, two. The total number of possible outcomes is $3 \times 3 \times 2 = 18$.

Bruce McAskill et al., *MathLinks 8: Teacher's Resource* (Toronto: McGraw-Hill Ryerson, 2008), 566. Reproduced by permission.

SC 5.

3.a.

		Spinner		
		1	2	3
Marble	G	G, 1	G, 2	G, 3
	R	R, 1	R, 2	R, 3
	B	B, 1	B, 2	B, 3
	Y	Y, 1	Y, 2	Y, 3

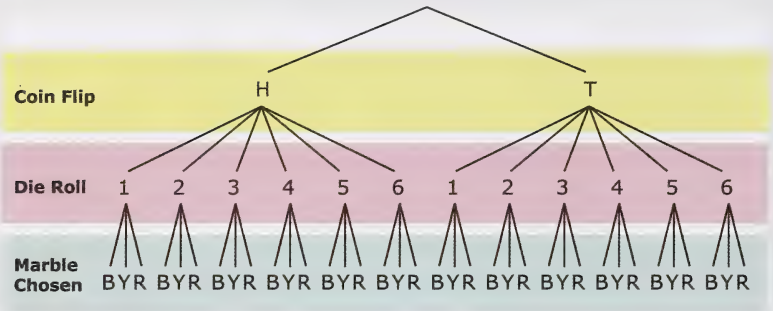
b. Count the outcomes in the table. There are 12 possible outcomes.

c. Use the multiplication method. There are four different marbles and three different regions on the spinner:

$3 \times 4 = 12$

There are 12 possible outcomes.

5.a. Start the tree diagram with the two outcomes from the coin flip. There are six possible outcomes for the roll of the die and three outcomes for the marble selection.

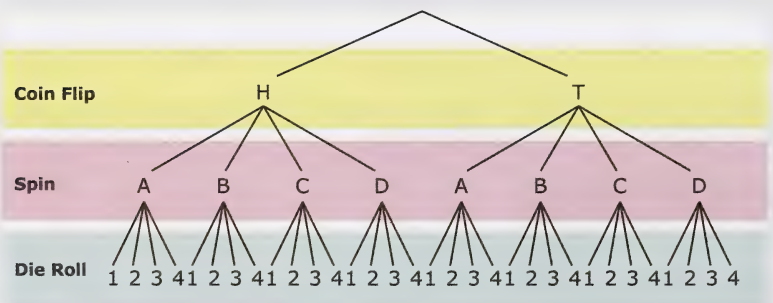


b. Count the bottom row of branches. There are 36 possible outcomes.

c. Use the multiplication method: $2 \times 6 \times 3 = 36$

There are 36 possible outcomes in this three-event probability experiment.

6.a. In the tree diagram there will be two outcomes for the coin flip, four outcomes for the spinner, and four outcomes for the roll of the die.



b. Count the bottom row of branches. There are 32 possible outcomes.

c. The total number of possible outcomes is $2 \times 4 \times 4 = 32$.

Bruce McAskill et al., *MathLinks 8: Solutions* CD (Toronto: McGraw-Hill Ryerson, 2008), 565-566. Reproduced by permission.

SC 6.

7. Using the multiplication method, the total shirt–pant combinations will be $4 \times 6 = 24$. Tony can make 24 shirt–pant combinations.

8. Count the number of pathways between Leftsville and Middletown: four.

Count the number of pathways between Middletown and Right City: three.

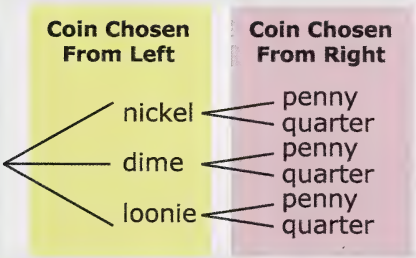
The total number of pathways from Leftsville to Right City is $4 \times 3 = 12$. There are 12 routes from Leftsville to Right City.

10.a. Use the multiplication method to find the number of coin combinations. She has three different coins in the left pocket and two different coins in the right pocket:

$3 \times 2 = 6$

She could get six combinations of coins.

b. A tree diagram confirms that there are six possible combinations.



c. The largest sum will consist of choosing the loonie and quarter:

$1.00 + 0.25 = 1.25$

The largest possible sum is $1.25.

d. The smallest sum will consist of choosing the nickel and penny:

$0.05 + 0.01 = 0.06$

The smallest possible sum is 6 cents or $0.06.

11. Answers will vary. Example: Jim has two pairs of shoes, four pairs of pants, and five dress shirts from which to choose. If he selects one item from each of the three types of clothing, how many combinations of clothing are possible?

$2 \times 4 \times 5 = 40$

There are 40 possible combinations of clothing.

Bruce McAskill et al., MathLinks 8: Solutions CD (Toronto: McGraw-Hill Ryerson, 2008), 567-568. Reproduced by permission.

Going Beyond

16. There are 26 choices of letters for each of the first three characters. There are eight possible digits for each of the last three characters:

$26 \times 26 \times 26 \times 8 \times 8 \times 8 = 8\,998\,912$

There are 8 998 912 possible car license plates.

Bruce McAskill et al., *MathLinks 8: Solutions* CD (Toronto: McGraw-Hill Ryerson, 2008), 569. Reproduced by permission.

Lesson 6

SC 1.

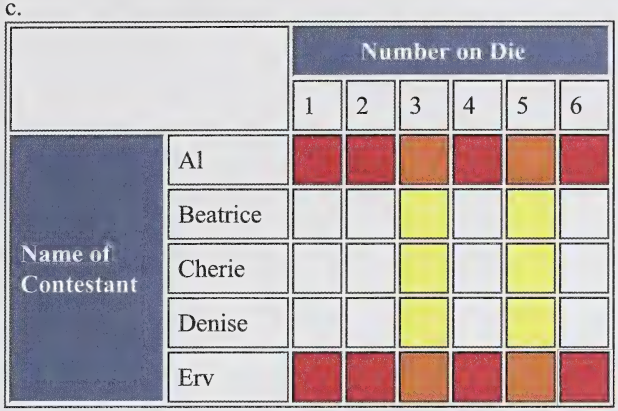
1.a. You should have made or printed a copy of the outcomes table as shown in 1.b.

b.

		Number on Die					
		1	2	3	4	5	6
Name of Contestant	Al						
	Beatrice						
	Cherie						
	Denise						
	Erv						

The fraction of the rows that are shaded is $\frac{2}{5}$, since 2 of the 5 rows are shaded (Al and Erv).

c.



Columns are vertical. Two columns, columns 3 and 5, are shaded out of a total of 6 columns. So the fraction of the columns that are shaded yellow is $\frac{2}{6}$.

2.a. The fraction of the total number of cells shaded red and yellow (resulting in orange) is $\frac{4}{30}$. This is found by counting the orange cells and counting total number of cells in the table (excluding the header cells).

b. $\frac{2}{5} \times \frac{2}{6} = \frac{4}{30}$

So, the fraction of the total number of cells shaded red and yellow (resulting in orange cells) can be found by multiplying the fractions for probabilities in 1.b. and 1.c.

c. The fraction $\frac{4}{30}$ represents the probability that Al or Erv will be chosen and will get to swim with dolphins or scuba dive on a coral reef. In other words, the fraction $\frac{4}{30}$ represents the successful outcome of the Wheel of Thrills contest.

Bruce McAskill et al., *MathLinks 8: Teacher's Resource* (Toronto: McGraw-Hill Ryerson, 2008), 573. Reproduced by permission.

SC 2.

a.
$$P(\text{blue} = 4) = \frac{1}{6}$$
$$P(\text{red} = 4) = \frac{1}{4}$$
$$P(\text{blue} = 4, \text{red} = 4) = P(\text{blue} = 4) \times P(\text{red} = 4)$$
$$= \frac{1}{6} \times \frac{1}{4}$$
$$= \frac{1}{24}$$

b. P (blue < 4)

There are three faces of the blue die that are less than four. So there are three favourable outcomes. There are a total of six faces, so there is a total of six possible outcomes.

$$P(\text{blue} < 4) = \frac{\text{favourable outcomes}}{\text{total possible outcomes}}$$
$$= \frac{3}{6}$$
$$= \frac{1}{2}(\text{expressed as a fraction in lowest terms})$$

P (red < 4)

There are three faces of the red die that are less than four. So there are three favourable outcomes. There are a total of four faces, so there is a total of four possible outcomes.

$$P(\text{red} < 4) = \frac{\text{favourable outcomes}}{\text{total possible outcomes}}$$
$$= \frac{3}{4}$$

$$P(\text{blue} < 4, \text{red} < 4) = \frac{3}{6} \times \frac{3}{4}$$
$$= \frac{9}{24}$$
$$= \frac{3}{8}$$

c.
$$P(\text{blue} = 4) = \frac{1}{6}$$
$$P(\text{red} < 4) = \frac{3}{4}$$
$$P(\text{blue} = 4, \text{red} < 4) = \frac{1}{6} \times \frac{3}{4}$$
$$= \frac{1}{8}$$

SC 3.

a. P (two boys) = $\frac{22}{100}$

The experimental probability of getting two boys is 0.22 or 22%.

b. The theoretical probability of getting one boy is P (one boy) = $\frac{1}{2}$. The theoretical probability of getting two boys is P (two boys) = $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$. The theoretical probability of getting two boys is 25% or 0.25.

c. The experimental probability of getting two boys is lower than the theoretical probability. The two probability values tend to differ when the number of trials is too small. The more trials you have, the closer the theoretical and experimental probabilities become.

SC 4.

4.a. There are six outcomes for the die and four regions of the spinner.

		Die					
		1	2	3	4	5	6
Spinner	A	A, 1	A, 2	A, 3	A, 4	A, 5	A, 6
	A	A, 1	A, 2	A, 3	A, 4	A, 5	A, 6
	B	B, 1	B, 2	B, 3	B, 4	B, 5	B, 6
	B	B, 1	B, 2	B, 3	B, 4	B, 5	B, 6

b. There are two cells in the table with favourable outcomes.

$$P(\text{A}, 2) = \frac{2}{24}$$
$$= \frac{1}{12}$$

c. The probability of spinning an A is $\frac{2}{4}$ or $\frac{1}{2}$. The probability of rolling a 2 is $\frac{1}{6}$. Therefore, the probability of spinning an A and rolling a 2 is $P(\text{A}, 2) = \frac{1}{2} \times \frac{1}{6} = \frac{1}{12}$.

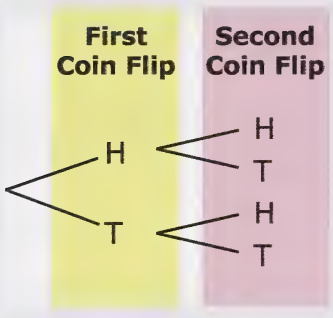
6.a. There are two equally likely outcomes: heads or tails.

$P(\text{H}) = \frac{1}{2}$

b. The probability that a head is flipped on both trials is $P(\text{H}, \text{H}) = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$.

c. Construct a tree diagram. The diagram shows that there are four possible outcomes and only one favourable outcome.

$P(\text{H}, \text{H}) = \frac{1}{4}$



8.a. Use two four-sided dice—or two spinners having four equal regions—to simulate the type of seed chosen and the location. Roll the two dice ten times and record the seed type and location in a tally chart. A possible experimental probability is $P(\text{marigold, flower pot}) = \frac{1}{10}$.

b. There are four equally likely types of flower seeds and four equally likely planting locations.

$$P(\text{marigold, flower plot}) = \frac{1}{4} \times \frac{1}{4}$$
$$= \frac{1}{16}$$

c. The experimental probability of $\frac{1}{10}$ is larger than the theoretical probability of $\frac{1}{16}$.

9.a. The red car was the last one collected during the simulation because there is only one tally mark in the chart.

b. The experimental probability for a blue car can be found by counting the favourable outcomes (4) and the total outcomes (13):

$$P(\text{blue}) = \frac{4}{13}$$
$$= 0.\overline{307\,692}$$

c. Determine the theoretical probability by counting the total number of equally likely outcomes: 5.

$$P(\text{blue}) = \frac{1}{5}$$
$$= 0.2$$

d. Use the multiplication method to find P (blue, blue):

$$P(\text{blue, blue}) = \frac{1}{5} \times \frac{1}{5}$$
$$= \frac{1}{25} \text{ or } 0.04$$

Bruce McAskill et al., *MathLinks 8: Teacher's Resource* (Toronto: McGraw-Hill Ryerson, 2008), 570-574. Reproduced by permission.

SC 5.

10. Use the multiplication method to determine the likelihood that it will rain in both cities.

$$P \text{ (Victoria rain and Calgary rain)} = \frac{75}{100} \times \frac{20}{100}$$

Note: Reducing the two fractions to lowest terms allows you to operate with smaller numbers.

$$= \frac{3}{4} \times \frac{1}{5}$$
$$= \frac{3}{20}$$

The probability that it will rain in both cities is $\frac{3}{20}$ or 15%.

11. There are five red balls in bag 1, which has a total of seven balls. There are six blue balls in bag 2, which has a total of ten balls.

$$P(\text{red, blue}) = \frac{5}{7} \times \frac{6}{10}$$
$$= \frac{30}{70}$$
$$= \frac{3}{7} \text{ or } 0.\overline{428\,571} \text{ or } 42.\overline{857\,142}\%$$

12.a. Spinner 1 must have three equal regions (in terms of area) that are labelled A, A, and B. Spinner 2 has four equal-sized regions that are labelled 2, 2, 2, and 3.

b. There are two favourable outcomes on spinner 1.

$$P(\text{A}) = \frac{2}{3}$$

c. There are three favourable outcomes on spinner 2.

$$P(2) = \frac{3}{4}$$

13.

a. There are four different pathways that lead from Lowtown to Highville.

b. Since the likelihood that a specific pumping station is working is $\frac{2}{3}$, assign numbers 1, 2, 3, and 4 on the die to simulate the pump is working, and numbers 5 and 6 to simulate the pump is not working. Roll the die ten times and determine the experimental probability that a specific pumping station is working.

c. There are seven trials where at least one pathway is available. The trials where there is not one available pathway are trials #4, #5, and #8.

$$P(\text{at least one pathway is available } \ldots) = \frac{7}{10}$$
$$= 0.7$$

The experimental probability that at least one pathway is available to carry water between the two towns is $\frac{7}{10}$ or 0.7.

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Going Beyond

17. First, find the probability of event A and event B occurring, $P(\text{A, B})$.

$$P(\text{A, B}) = P(\text{A}) \times P(\text{B})$$
$$= \frac{1}{2} \times \frac{3}{7}$$
$$= \frac{3}{14}$$

Then, find $P(\text{C})$.

$$P(\text{A, B, C}) = P(\text{A, B}) \times P(\text{C})$$
$$\frac{9}{70} = \frac{3}{14} \times P(\text{C}) \qquad \text{Substitute with known values.}$$
$$\frac{9}{70} \div \frac{3}{14} = P(\text{C}) \qquad \text{Change to a division statement.}$$
$$\frac{9}{70} \times \frac{14}{3} = P(\text{C})$$
$$\frac{3}{5} = P(\text{C})$$

The probability of event C is $\frac{3}{5}$.

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Unit Summary

SC 1.

1.a. The favourite type of music was rap. The least favourite type of music was classical.

2.a. The sales for DVDs in 2006 were $18 000.

b. CD sales dropped from 152 000 in 2006 to 120 000 in 2007.

c. Over time, CD and DVD sales are likely to continue to drop, since the second bar in each set is shorter than the first one. This trend is consistent with the growing popularity of online sources for music and movies.

3.b. Your answers may differ from those of others. Be sure to explain your choice. Sample answer: I prefer the circle graph. It lets a person compare the percent of each producer's market share to the entire market.

Bruce McAskill et al., *MathLinks 8: Teacher's Resource* (Toronto: McGraw-Hill Ryerson, 2008), 9. Reproduced by permission.

SC 2.

1. The word *likelihood* refers to the chance or probability that you will win the game.

2. Card games may include Go Fish and Crazy Eights. Games that involve dice may include Backgammon, Yahtzee®, and Game of Life®.

3. Sample answers include the following:

    - In Backgammon, the throw of the dice determines how far you advance the pieces.

    - In the Game of Life®, the outcome of rolling the dice determines what type of job (and salary) you will have and how many kids you will have in your family. These outcomes will affect the amount of money you earn during the game and your likelihood of retiring in style and winning the game

Adapted from Bruce McAskill et al., *MathLinks 8: Teacher's Resource* (Toronto: McGraw-Hill Ryerson, 2008), 552. Reproduced by permission.

SC 3. Turn to pages 481 and 482 of the textbook. Check your answers with those provided.

SC 4. Turn to page 512 of the textbook. Check your answers with those provided.

SC 5.

1. B

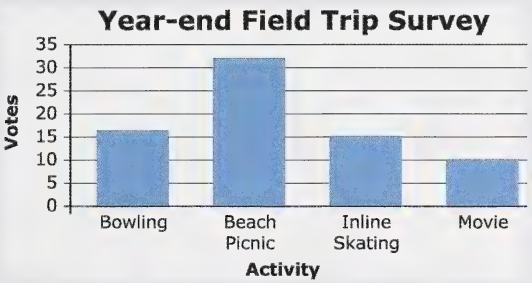
A circle graph is best used for comparing each category of data with the whole.

3. C

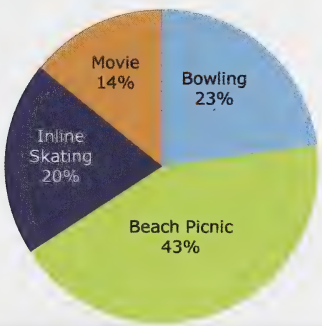
The graph is misleading because the width of the bars varies, making it appear that the change in cost is greater than it actually is.

5. He should use a line graph. A line graph is used to show changes over time.

7. a.

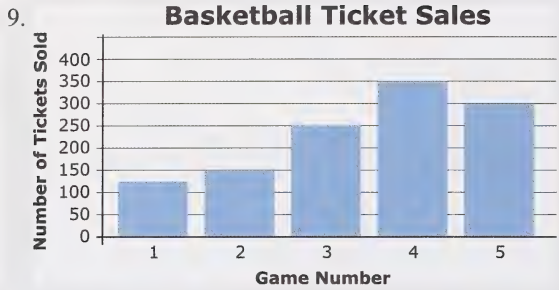


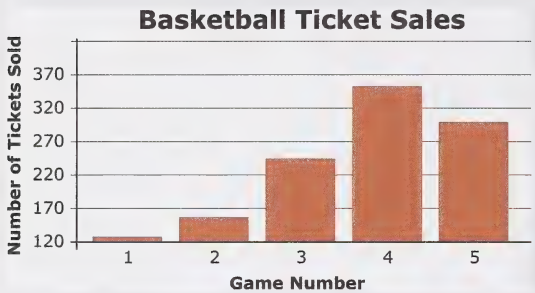
b. **Year-end Field Trip Study (74 students)**



c. Answers may vary. Example: The bar graph displays the number of votes for each activity, so it is very clear that the beach picnic is the favourite activity. The circle graph displays the percent of votes for each activity. It also makes it very clear that the favourite activity is the beach picnic.

9.





The second graph is misleading because the vertical axis starts at 120. This makes it appear that there were almost five times as many tickets sold for game 4 than for game 1. The actual number of tickets sold for game 4 was 350 tickets, and for game 1 it was 125 tickets. These amounts show that there were actually $350 \div 125 = 2.8$ times as many tickets sold for game 4.

Adapted from Bruce McAskill et al., *MathLinks 8: Teacher's Resource* (Toronto: McGraw-Hill Ryerson, 2008), 37-38. Reproduced by permission.

SC 6.

1. A

There are 16 possible outcomes. Four outcomes are favourable.

$$P(\text{same number on both die}) = \frac{4}{16}$$
$$= \frac{1}{4}$$

3. A

There are nine favourable outcomes.

$$P(\text{neither die has a two showing}) = \frac{9}{16}$$

5. The probability of the coin landing heads up is $\frac{1}{2}$. The probability that the spinner stops on a vowel is $\frac{2}{6}$ or $\frac{1}{3}$.

P (the coin lands heads up and the spinner stops on a vowel) $= \frac{1}{2} \times \frac{1}{3}$

7.a. There are two red jellybeans and a total of five jellybeans in the jar.

P (red jellybean) $= \frac{2}{5}$ or 40%

b. There are five orange marbles and a total of eight marbles in the bag.

P (orange marble) $= \frac{5}{8}$

There are three blue jellybeans in the jar.

P (blue jellybean) $= \frac{3}{5}$

The probability of selecting an orange marble and a blue jellybean is as follows:

$$P(\text{orange marble, blue jellybean}) = \frac{5}{8} \times \frac{3}{5}$$

$$= \frac{15}{40}$$

$$= \frac{3}{8} \text{ or } 0.375$$

9.a. $P(\text{rain on Saturday, rain on Sunday}) = \frac{7}{10} \times \frac{1}{2}$

$$= \frac{7}{20} \text{ or } 35\%$$

There is a 35% chance that it will rain on both Saturday and Sunday.

b. The probability that it will not rain on Saturday is as follows:

$$P(\text{no rain on Saturday}) = \frac{7}{10}$$
$$= \frac{10}{10} - \frac{7}{10}$$
$$= \frac{3}{10} \text{ or } 30\%$$

$$P(\text{no rain on Sunday}) = 1 - \frac{1}{2}$$
$$= \frac{1}{2}$$

$$P(\text{no rain on Saturday, no rain on Sunday}) = \frac{3}{10} \times \frac{1}{2}$$
$$= \frac{3}{20} \text{ or } 15\%$$

c. Answers will vary. Example: Jamie could mark seven cards with R for rain and leave the other three cards blank. She should shuffle the cards and then randomly pick one card to simulate whether it will rain on Saturday. She should then take one blank card and one card marked R and randomly select one of these two cards to see if it will rain on Sunday.

d. Answers will vary. Example: In general, the more trials that are performed, the more closely the experimental results approximate the theoretical results. One hundred trials will likely be more accurate than ten trials.

Bruce McAskill et al., *MathLinks 8: Teacher's Resource* (Toronto: McGraw-Hill Ryerson, 2008), 587-589. Reproduced by permission.

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